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THESIS

AMPLITUDE SHADING AND PHASE WEIGHTING OF A VERTICAL LINEAR ARRAY IN THE SOFAR CHANNEL BY THE LINEAR MINIMUM VARIANCE ESTIMATION TECHNIQUE

by

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Amplitude Shading and Phase Weighting of a Vertical Linear Array in the SOFAR Channel by the Linear Minimum Variance Estimation Technique

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I. INTRODUCTION

There are many sclutions to the problem of predicting the location of an underwater energy source. One common solution is the use cf a passive hydrophone which detects the pressure waves radiating from the source. The hydrophone sensor is assumed to be omnidirectional and therefore incapable of estimating direction. To provide directionality a series of sensors are placed in a row to form a passive linear array.

A familiar methcd of determining directionality is time-domain beamforming. In this principle, it is assumed that the source is far enough away so that the pressure wave appears to be a plane wave when viewed at the site of the receiving array.

Thus a set of time delays are calculated for any direction cf signal arrival, which, when applied to the receiver outputs causes them to be in phase and to reinforce when summed. The resultant angular response to signals arriving from other than the nominated direction is then a function cf the array geometry, relative to the signal wavelength, and any weighting factors which have been applied to the receiver outputs. The effect is to generate a main receiving beam in the desired direction, with a series of undesired subsidiary sidelobes whose magnitude can be controlled to some extent by the choice of a suitable array geometry and the use cf amplitude weightings on the receiver outputs.

In crder to determine location, three or more such arrays separated by a known amount may be used.

This study is ccncerned with a single linear vertical passive array and the determination of the depth of a single

underwater source. The analysis is based on the following assumptions:

- The underwater source is emitting continuously and at a monochromatic frequency.
- Both the source and receiving hydrophones are stationary in space causing a constant range.
- The range is sufficiently long so that the channel is filled with R-R (refracted-refracted) rays.
- There is no distortion introduced in the propagating medium so that the signals received at each sensor are identical except for constant delays.
- The source signal and noise are independent and staticnary gaussian random processes.
- The speed of sound profile is triangular and symmetric with the deep sound channel axis at 1000 meters. The velocity gradient is -0.017 meters/meter/sec above 1000 meters and +0.017 meters/meter/sec below 1000 meters. This profile gives a speed of sound at the surface of 1500 meters/sec.
- The speed of sound profile is constant in the horizontal plane.
- Only R-R rays are considered. All other rays have sufficient loss that their effect is negligible.

As opposed to conventional time-domain beamforming, this study makes no assumption of planar wave fronts at the receiver site. Therefore the time delays applied to each receiver will not, in general, be a linear function of depth.

Since it is desired to determine whether or not there is a source present at a specific depth the result will be a

binary decision. A "1" will indicate signal source present; a "0" will indicate signal source not present. For the constant range there will be "N" test depths investigated for the signal source. The number of hydrophones in the vertical array will be "L".

For a single source at a given depth, the travel time is calculated from the depth to each hydrophone. This travel time is converted into a phase delay for each hydrophone so that after summation from all hydrophones a maximum output is achieved. This output is then passed through a squaring device, an integrator, and a threshold and flip flop device to give a "1" binary output. If the signal source is at a different depth and the same previous phase delays are used for each hydrophone then the output will be somewhat less than the previous maximum. The difference in depth required to achieve a "0" binary output is the depth resolution of the system.

The travel times for each hydrophone are calculated for each of the "N" source depths to be considered. "N" will ordinarily be much greater than "L" so that the system will be overdetermined. An overdetermined system is one in which there are more equations than unknowns. The objective then is to calculate the phase angle and amplitude weight for each hydrophone so that a determination can be made indicating the presence or absence of a signal source at a given depth.

The method used to calculate the phase and amplitude weights is the linear minimum variance estimation technique. Linear minimum variance estimators are optimum when compared with all other estimators for gaussian problems. The method is directly applicable to overdetermined systems.

The output of the summer is calculated using the linear minimum variance amplitude and phase angles assuming a source at one of the "N" source depths and no source at the

others. The calculation is repeated for each of the depths. The result, when plotted against source depth, will be referred to as the "beam pattern" of the array in this report. (Although similar, it should not be interpreted as the angular response of an array as in the conventional definition of a beam pattern. The conventional definition loses much of its utility when the wavefronts are not planar.) Ideally the beam pattern will be maximum at the desired depth and very small at all other depths so that the binary "1" decision will be made for a source at the desired depth, and a "0" decision for sources at all others. This beam pattern is compared with the depth beam pattern of the conventional time-domain beamformer mentioned above. The purpose of this thesis is to determine, as an initial investigation, whether the linear minimum variance estimation technique, when applied to a linear vertical array, is useful in depth discrimination at long ranges in a 'SOFAR' type sound channel.

II. GENERAL THEORY

A. RAY ACOUSTICS

The propagation of sound in an elastic medium can be described mathematically by solutions of the wave equation using the appropriate boundary and medium conditions for a particular problem. The wave equation relating the acoustic pressure 'p' to the coordinates 'x','y','z', and the time 't', may be written as

$$\frac{d^2p}{dt^2} = c^2 \frac{d^2p}{dx^2} + \frac{d^2p}{dy^2} + \frac{d^2p}{dz^2} \qquad (2.1)$$

where 'c' is a quantity that has the general significance of sound velocity and may vary with the coordinates.

One may approximate the solution of the wave equation using ray theory: its body of results and conclusions is called ray acoustics.

Officer [Ref. 1] describes the ray solution as a complete solution to any particular propagation problem within the validity of the approximation of the Eikonal equation to the wave equation. For these approximations to be valid neither the amplitude of the wave nor the speed of sound can change appreciably in distances comparable to a wavelength.

Thus the path of a ray through a medium in which the speed of sound varies with depth can be calculated by the application of Snell's law

$$\cos\theta/c = 1/c_0 = \text{a constant for any one ray} \qquad (2.2)$$

where 'θ' is the angle of depression made with the horizontal at a depth where the speed of sound is 'c', and 'c_0' is the speed at a depth (real or extrapolated) where the ray would become horizontal.

In a medium in which the velocity of sound changes linearly with depth the sound rays can be shown to be arcs of circles, that is, to have a constant radius of curvature. Kinsler et al. [Ref. 2] give a simple and heuristic demonstation of the circularity of rays in a medium with a linear sound speed gradient 'g'. The center of the circle which creates the arc lies at a depth where the sound speed extrapolates to zero. To understand this, consider a portion of a ray path with a local radius of curvature 'R', as illustrated in Figure 2.1. Since the gradient 'g' for this case is

$$g = \Delta c/\Delta z = (c_2-c_1)/(d_2-d_1) = (c_2-c_1)/R(\cos\theta_1-\cos\theta_2) \qquad (2.3)$$

where 'Δc' is the change in sound speed and 'Δz' is the change in depth. It can be seen that the radius of curvature is given by

$$R = -c_0/g = -c/(g\ \cos\theta) \qquad (2.4)$$

The ray path is therefore a circle when 'g' is constant because 'R' is then constant. The center of curvature of a circle lies at the depth where θ is 90 degrees, which corresponds to c=0. For the situation in Figure 2.1 the speed gradient is negative so that 'R' is positive. If the speed gradient were positive 'R' would be negative, and the path would curve upward.

Once the radius of curvature of each segment of a path is known the actual path can be traced graphically or

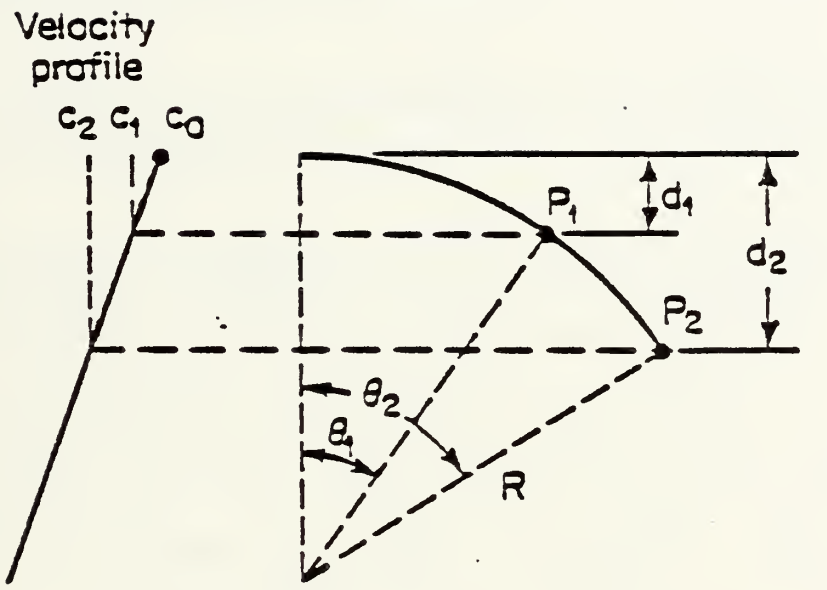

Figure 2.1 Circular Ray Path

computed. If the initial angle of depression of a ray is $\theta 1$, then by referring to the geometry of Figure 2.1 the changes in both range and depth are

$$\Delta r = c_1 (\sin(\theta 1) - \sin(\theta 2))/(g \cos(\theta 1)) \qquad (2.5)$$

$$\Delta z = c_1 (\cos(\theta 2) - \cos(\theta 1))/(g \cos(\theta 1)) \qquad (2.6)$$

The sign convention for these equations is: downward, to the right, and depression angles below the horizontal axis positive.

The symmetric triangular sound speed profile assumed in the introduction is similar to speed profiles encountered in the deep sound channel, sometimes called the SOFAR channel. The velocity minimum which occurs at the axis of the sound channel causes the sea to act like a kind of lens; above and below the minimum, the velocity gradient continually bends the sound rays toward the depth of minimum velocity. A portion of the power radiated by a source in the deep sound channel accordingly remains within the channel and encounters no acoustic losses by reflection from the surface and bottom. These rays are called R-R (refracted-refracted) and since they have very low transmission loss, very long ranges can be obtained from a source of moderate acoustic power output. Thus an energy source at a specific depth will propagate energy in all directions but only the direction which is toward the receiving array and for which the rays are R-R is of interest. To determine the range of depression angles which will yield R-R rays Snell's law is used to give

$$\theta max = \arccos(c1/(c1-(g\ d))) \qquad (2.7)$$

where 'θmax' is in radians and 'c1' is the speed of sound at the source depth 'd'.

An example is given in Figure 2.2 of a single ray trace propagating in the SOFAR channel to show how equations 2.5 and 2.6 are used in determining range and depth. The ray path is broken up into arcs of circles as shown in the figure, and then by paying close attention to the previously defined sign conventions, the change in range and depth is found. Being more specific, for arc 1, $\theta 1$ and $\theta 2$ are both positive; for arc 2, $\theta 1$ is positive and $\theta 2$ is zero; for arc 3, $\theta 1$ is zero and $\theta 2$ is negative; and for arc 4, $\theta 1$ is negative and $\theta 2$ is zero. By keeping a running total of all the depth and range changes it is possible to determine the total horizontal distance travelled and the depth at that distance.

For the speed of sound profile assumed in the introduction a computer generated ray plot is shown in Figure 2.3 for a source depth of 300 meters. As each ray propagates out from the source the triangular channel becomes filled with sound. If a receiving hydrophone is placed a great distance away, a number of refracted propagation paths will exist, each having a different travel time and crossing the channel axis at different intervals. The path with the greatest excursion from the axis will have the shortest travel time.

Officer [Ref. 1] shows that the travel time 't' of a ray, which is an arc of a circle, is given by

$$t = \frac{1}{g}\int_{\theta_1}^{\theta_2} \frac{d\theta}{\cos\theta} \qquad (2.8)$$

and the travel time for each arc is

$$t = -\frac{1}{g} \log_e \left[\frac{\tan((\pi/4)+(\theta_2/2))}{\tan((\pi/4)+(\theta_1/2))} \right] \qquad (2.9)$$

Equation 2.9, applied using the same convention as equations 2.5 and 2.6, determines the total travel time of a ray in the deep sound channel.

B. ARRAY MODEL

The receiving linear vertical array is presumed to consist of 'L' hydrophones as shown in Figure 2.4. It is assumed that the source is emitting energy at a constant frequency, 'f', and amplitude, 'A', regardless of the depth. The source signal at the source is $A\exp(j2\pi ft)$. The inherent received signal at the first hydrophone is

$$x1(t) = A'\exp(j2\pi f(t-t1)) \qquad (2.10)$$

where 't1' is the travel time from the energy source to the first hydrophone and 'A'' is the amplitude of the signal at the range of the array. After passage through the amplitude weight 'a1' and a phase delay of 'τ_1', the signal on the first hydrophone at the input to the summer is

$$y1(t)=A'a1x1(t-\tau_1)=A'a1\exp(j2\pi f(t-t1-\tau_1)) \qquad (2.11)$$

A time delay, for monochromatic signals corresponds to a phase shift

$$\theta = 2\pi ft \qquad (2.12)$$

where 'θ' is the phase shift in radians and 't' is the time delay in seconds. Thus equation 2.11 can be written as

$$y1(t) = A' a1 \exp(j(2\pi ft - \phi 1 - \theta 1)) \qquad (2.13)$$

where '$\phi 1 = 2\pi ft1$' is the phase delay due to the travel time from the source to the first hydrophone and '$\theta 1$' is the phase delay in the receiver on the first hydrophone.

Combining all the hydrophones in the array in a summer gives as an expression for the array output

$$Y(t) = \sum_{k=1}^{L} A' a_k \exp(j(2\pi ft - \phi_k - \theta_k)) \qquad (2.14)$$

where 'ϕ_k' represents the phase delay due to the travel time from the source to the "k th" hydrophone and 'θ_k' is the phase delay in the receiver on the "k th" hydrophone. If the amplitude of the energy source is normalized by setting A'=1, and equation 2.14 is written in terms of real and imaginary components, we have

$$Y(t) = \sum_{k=1}^{L} a_k \cos(-\phi_k - \theta_k) + j\sin(-\phi_k - \theta_k) \exp(j2\pi ft) \qquad (2.15)$$

When an energy source is at the "q th" depth, we wish to have each receiving hydrophone's phase delay cancel out the effect of the travel time from the source to it, such that '$-\phi_k - \theta_k$' is equal to a multiple of '2π'. This will put all signals into the summer in phase and thus maximize the signal gain for a source at the "q th" depth. From equation 2.15,

$$\sum_{k=1}^{L} (a_{kq} \cos(-\phi_{kq} - \theta_{kq})) = L \quad \text{(source present at q)} \qquad (2.16)$$

$$\sum_{k=1}^{L} (a_{kq} \sin(-\phi_{kq} - \theta_{kq})) = 0 \quad \text{(source present at q)} \qquad (2.17)$$

Note that the first subscript on the phase angle indicates the receiving hydrophone and the second subscript indicates the depth of the source. Thus 'ϕ_{kq}' would indicate the phase shift relating to the travel time from the "q th" test depth to the "k th" hydrophone in the receiving array.

It is desirable for 'Y(t)' to be a minimum value for sources at other than the depth being investigated. Thus for each of the other 'N-1' depths 'Y(t)' is set to zero. This gives 'N-1' equations for the real terms of 'Y(t)' set to zero

$$\sum_{k=1}^{L} a_{kq}\cos(-\phi_{km}-\theta_{kq})=0 \quad \text{(source absent; } m\neq q) \tag{2.18}$$

and 'N-1' equations for the imaginary terms of 'Y(t)' set to zero

$$\sum_{k=1}^{L} a_{kq}\sin(-\phi_{km}-\theta_{kq})=0 \quad \text{(source absent; } m\neq q) \tag{2.19}$$

By using elementary trigonometric identities, equation 2.16 (real terms with source present at "q th" depth) becomes

$$\sum_{k=1}^{L} a_{kq}\left[\cos(\phi_{kq})\cos(\theta_{kq})-\sin(\phi_{kq})\sin(\theta_{kq})\right]=L \tag{2.20}$$

Equation 2.18 (real terms with source absent for each of the other 'N-1' depths) becomes

$$\sum_{k=1}^{L} a_{kq}\left[\cos(\phi_{km})\cos(\theta_{kq})-\sin(\phi_{km})\sin(\theta_{kq})\right]=0 \quad m=1,2,\ldots N;\ m\neq q \tag{2.21}$$

Equation 2.17 (imaginary terms with source present at the "q th" depth) becomes

$$\sum_{k=1}^{L} -a_{kq}(\sin(\phi_{kq})\cos(\theta_{kq})+\cos(\phi_{kq})\sin(\theta_{kq}))=0 \qquad (2.22)$$

and equation 2.19 (imaginary terms with source absent for each of the other 'N-1' depths) becomes

$$\sum_{k=1}^{L} -a_{kq}\left[\sin(\phi_{km})\cos(\theta_{kq})+\cos(\phi_{km})\sin(\theta_{kq})\right]=0 \quad m=1,2,\ldots N;\ m\neq q \qquad (2.23)$$

Thus, there are a total of '2N' equations with '2L' unknowns.

In order to simplify, we put these '2N' equations into matrix form. Arbitrarily the real terms are made the first 'N' equations and the imaginary terms the second 'N' equations. The first real and first imaginary equation is at the lowest (shallowest) source depth and equations increase in order after that until the last real and last imaginary equation correspond to a source at the deepest depth. The resultant matrix equation becomes:

$$\begin{bmatrix}0\\0\\\vdots\\0\\L\\0\\\vdots\\0\\\hline 0\\0\\\vdots\\0\end{bmatrix} = \left[\begin{array}{cccc|cccc} \cos\phi_{11} & \cos\phi_{21} & \cdots & \cos\phi_{L1} & -\sin\phi_{11} & -\sin\phi_{21} & \cdots & -\sin\phi_{L1} \\ \cos\phi_{12} & \cos\phi_{22} & \cdots & \cos\phi_{L2} & -\sin\phi_{12} & -\sin\phi_{22} & \cdots & -\sin\phi_{L2} \\ \vdots & & & & \vdots & & & \\ \cos\phi_{1q} & \cos\phi_{2q} & \cdots & \cos\phi_{Lq} & -\sin\phi_{1q} & -\sin\phi_{2q} & \cdots & -\sin\phi_{Lq} \\ \vdots & & & & \vdots & & & \\ \cos\phi_{1N} & \cos\phi_{2N} & \cdots & \cos\phi_{LN} & -\sin\phi_{1N} & -\sin\phi_{2N} & \cdots & -\sin\phi_{LN} \\ \hline -\sin\phi_{11} & -\sin\phi_{21} & \cdots & -\sin\phi_{L1} & -\cos\phi_{11} & -\cos\phi_{21} & \cdots & -\cos\phi_{L1} \\ -\sin\phi_{12} & -\sin\phi_{22} & \cdots & -\sin\phi_{L2} & -\cos\phi_{12} & -\cos\phi_{22} & \cdots & -\cos\phi_{L2} \\ \vdots & & & & \vdots & & & \\ -\sin\phi_{1N} & -\sin\phi_{2N} & \cdots & -\sin\phi_{LN} & -\cos\phi_{1N} & -\cos\phi_{2N} & \cdots & -\cos\phi_{LN} \end{array}\right] \begin{bmatrix} a_{1q}\cos\theta_{1q} \\ a_{2q}\cos\theta_{2q} \\ \vdots \\ a_{Lq}\cos\theta_{Lq} \\ \hline a_{1q}\sin\theta_{1q} \\ a_{2q}\sin\theta_{2q} \\ \vdots \\ a_{Lq}\sin\theta_{Lq} \end{bmatrix} \qquad (2.24)$$

Simplifying further, equation 2.24 becomes:

$$\begin{bmatrix} 0 \\ \vdots \\ L \\ \vdots \\ 0 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} \underline{\alpha} & -\underline{\beta} \\ \hline -\underline{\beta} & -\underline{\alpha} \end{array}\right] \left[\begin{array}{c} \underline{c} \\ \hline \underline{s} \end{array}\right] \tag{2.25}$$

where '$\underline{\alpha}$', '$\underline{\beta}$', '$\underline{c}$', and '$\underline{s}$' represent the appropriate submatrices.

Then, by noting that the multiplication of each element of a column by the same nonzero constant doesn't affect the solution, equation 2.25 becomes:

$$\begin{bmatrix} 0 \\ \vdots \\ L \\ \vdots \\ 0 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} \underline{\alpha} & \underline{\beta} \\ \hline -\underline{\beta} & \underline{\alpha} \end{array}\right] \left[\begin{array}{c} \underline{c} \\ \hline \underline{s} \end{array}\right] \tag{2.26}$$

Finally letting $\underline{Z}$, $\underline{A}$, and $\underline{\theta}$ represent the matrices in equation 2.26, we obtain

$$\underline{Z} = \underline{A}\underline{\theta} \tag{2.27}$$

where a matrix is denoted by a capitalized underlined letter.

In summary, $\underline{Z}$ is the desired response of the vertical array to the 'N' source test depths. $\underline{A}$ represents known travel times from each of the 'N' source depths to each of the 'L' receiving hydrophones. $\underline{\theta}$ is unknown. It is the phase and amplitude weighting which must be applied to the vertical array in order to realize $\underline{Z}$. $\underline{\theta}$ contains '2L' unknowns.

Equation 2.27 represents '2N' equations. Since this system of equations is overdetermined (N>L) an exact solution does not exist. In order to make the best estimate of $\underline{\theta}$ for the desired response, the linear minimum variance estimation technique is used.

C. LINEAR MINIMUM VARIANCE METHOD

Equation 2.27 represents a noise free environment. If noise were present it would become

$$\underline{Z} = \underline{A\theta} + \underline{n} \tag{2.28}$$

with '$\underline{n}$' a "2N" element column vector. This represents the noise at each source depth. '$\underline{Z}$' is a linear function of '$\underline{\theta}$' and is called the oberservation. '$\underline{A}$' is a "2Nx2L" modulation or observation matrix which is known, and '$\underline{\theta}$' is the "2L" element random parameter vector which is to be estimated. Assume the first and second moments of '$\underline{\theta}$' and '$\underline{n}$' are given by

$$E(\underline{\theta}) = \underline{\mu}_\theta \qquad Var(\underline{\theta}) = \underline{V}_\theta \tag{2.29}$$

and

$$E(\underline{n}) = \underline{0} \qquad Var(\underline{n}) = \underline{V}_n \tag{2.30}$$

where 'E' represents expectation or first moment and 'Var' represents the covariance matrix. It is assumed that the parameter '$\underline{\theta}$' and the noise '$\underline{n}$' are uncorrelated.

A restriction imposed is that the estimate must be a weighted linear combination of the observations:

$$\hat{\underline{\theta}}_L = \underline{b} + \underline{B}\underline{Z} \tag{2.31}$$

where '^' indicates estimate. The objective is to select '$\underline{b}$' and '$\underline{B}$' in order to minimize the error variance. Such an estimator is the linear minimum variance estimator; it is the best, in the sense of minimum-error-variance linear estimators.

Another restriction is that the estimator be unbiased; in otherwords it is required that the expected value of the estimator '$\hat{\underline{\theta}}_L$' is equal to the expected value of the parameter '$\underline{\theta}$'. Thus,

$$E(\hat{\underline{\theta}}_L) = \underline{b} + \underline{B}E(\underline{z}) = E(\underline{\theta}) = \underline{\mu}_\theta \tag{2.32}$$

yielding

$$\underline{b} = \underline{\mu}_\theta - \underline{B}\underline{A}\underline{\mu}_\theta \tag{2.33}$$

Substituting this result in equation 2.31 gives for the unbiased linear estimator

$$\hat{\underline{\theta}}_L = \underline{\mu}_\theta + \underline{B}(\underline{Z} - \underline{A}\underline{\mu}_\theta) \tag{2.34}$$

Note that since the estimator is unbiased, the estimation error '$\underline{\theta}_E = \underline{\theta} - \hat{\underline{\theta}}_L$' is zero mean. The next step is to select '$\underline{B}$' in order to minimize the error variance. However, this optimization problem is ill-defined because the error variance is a matrix. Therefore in order to introduce a scalar goodness measure the sum of the variances of each component of '$\underline{\theta}$' is minimized. This is the sum of the main diagonal terms of the covariance matrix and is defined as the trace of the matrix.

$$\text{tr}\,(\text{Var}(\underline{\theta}_E)) = \sum_{n=1}^{2N} \text{Var}((\underline{\theta}_E)_n) \tag{2.35}$$

where 'tr' indicates trace. '$\underline{B}$' is then selected to minimize the trace of the error variance, or

$$\min_B \text{tr}\,(\text{Var}(\underline{\theta}_E)) = \min_B \text{tr}\,(E(\underline{\theta}_E \underline{\theta}_E^T)) \tag{2.36}$$

where '^{T}' indicates the transpose. The following problem is then obtained by substituting equation 2.34 into equation 2.36:

$$\min_B \text{tr}\,(\text{Var}(\underline{\theta}_E)) = \min_B \text{tr}(E((\underline{\theta}-\underline{u}_\theta-\underline{B}(\underline{Z}-\underline{Au}_\theta)(\underline{\theta}-\underline{u}_\theta\underline{B}(\underline{Z}-\underline{Au}_\theta)^T)) \tag{2.37}$$

It is well known [Ref. 3] that equation 2.37 is minimized when

$$\text{Cov}(\underline{\theta},\underline{Z}) - \underline{B}\text{Var}(\underline{Z}) = \underline{0} \tag{2.38}$$

where '$\text{Cov}(\underline{\theta},\underline{Z})$' is the covariance matrix of the unknown parameters and the observations. Denoting the optimum filter by '$\underline{B}^*$', then if

$$\underline{B}^* = \text{Cov}(\underline{\theta},\underline{Z})(\text{Var}(\underline{Z}))^{-1} \tag{2.39}$$

a minimum is achieved for the sum of the squares of the errors.

Using equation 2.28 for '$\underline{Z}$', the covariance of '$\underline{\theta}$' and '$\underline{Z}$' becomes

$$\text{Cov}(\underline{\theta},\underline{Z}) = \text{Cov}(\underline{\theta},\underline{A\theta}+\underline{n}) = \underline{V}_\theta \underline{A}^T \tag{2.40}$$

since '$\underline{\theta}$' and '$\underline{n}$' are uncorrelated. The variance of '$\underline{Z}$' is

$$\text{Var}(\underline{Z}) = \text{Var}(\underline{A}\underline{\theta}+\underline{n}) = \underline{A}\underline{V}_{\theta}\underline{A}^{T}+\underline{V}_{n} \tag{2.41}$$

Substituting these into equation 2.39 gives

$$\underline{B}^{*} = \underline{V}_{\theta}\underline{A}^{T}(\underline{A}\underline{V}_{\theta}\underline{A}^{T}+\underline{V}_{n})^{-1} \tag{2.42}$$

and the linear minimum variance estimator is

$$\underline{\hat{\theta}}_{LMV} = \underline{\mu}_{\theta}+\underline{V}_{\theta}\underline{A}^{T}(\underline{A}\underline{V}_{\theta}\underline{A}^{T}+\underline{V}_{n})^{-1}(\underline{Z}-\underline{A}\underline{\mu}_{\theta}) \tag{2.43}$$

By utilizing a matrix inversion lemma [Ref. 3] equation 2.43 becomes

$$\underline{\hat{\theta}}_{LMV} = (\underline{A}^{T}\underline{V}_{n}^{-1}\underline{A}+\underline{V}_{\theta}^{-1})^{-1}(\underline{A}^{T}\underline{V}_{n}^{-1}\underline{Z}+\underline{V}_{\theta}^{-1}\underline{\mu}_{\theta}) \tag{2.44}$$

The advantage of this equation over equation 2.43 is the size of the matrix to be inverted. In equation 2.43 the matrix has dimensionality '2N' while in equation 2.44 its dimenionality is only '2L'. Thus the advantages of the linear variance estimator are the ease with which they are derived, the mathematical tractability of the linear form, and the minimum amount of stochastic information required for development. An interesting characteristic is that the linear minimum variance estimate is the orthogonal projection of '$\underline{\theta}$' onto the space spanned by the observation '$\underline{Z}$'. Because of these factors this estimator is a popular form for estimating unknowns in overdetermined equations.

For this thesis it is assumed that the noise samples are uncorrelated and identically distributed so that:

$$\underline{V}_{n} = \sigma^{2}\underline{I} \tag{2.45}$$

No previous knowledge is assumed about 'θ'. This implies an infinite variance matrix which is represented as:

$$\underline{V}_{\theta}^{-1}=\underline{0} \qquad \text{and} \qquad \underline{\mu}_{\theta}=\underline{0} \tag{2.46}$$

The linear minimum variance estimate given by equation 2.44 is then

$$\underline{\hat{\theta}}_{LMV}=(\underline{A}^{T}\underline{A})^{-1}\underline{A}^{T}\underline{Z} \tag{2.47}$$

By determining '$\underline{\hat{e}}_{LMV}$' the phase and amplitude weights are found for a signal source on the 'q th' depth. Recall that

$$\underline{\hat{\theta}}_{LMV} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \vdots \\ \hat{\theta}_L \\ \hat{\theta}_{L+1} \\ \vdots \\ \hat{\theta}_{2L} \end{bmatrix} = \begin{bmatrix} a_{1q}\cos\theta_{1q} \\ a_{2q}\cos\theta_{2q} \\ \vdots \\ a_{Lq}\cos\theta_{Lq} \\ a_{1q}\sin\theta_{1q} \\ \vdots \\ a_{Lq}\sin\theta_{Lq} \end{bmatrix} \tag{2.48}$$

Upon solving, this equation gives for the phase delay

$$\hat{\theta}_{mq}=\arctan(\hat{\theta}_{L+m}/\hat{\theta}_{m}) \quad \text{where } m=1 \text{ to } L \tag{2.49}$$

and amplitude weight

$$\hat{a}_{mq}=\hat{\theta}_{m}/\cos(\hat{\theta}_{mq}) \quad \text{where } m=1 \text{ to } L \tag{2.50}$$

When these amplitude weights and phase delays are applied to the vertical linear array a resulting beam pattern is formed which in the absence of noise is:

$$\hat{\underline{Z}} = \underline{A}\hat{\underline{\theta}} \qquad (2.51)$$

The resulting beam pattern '$\hat{\underline{Z}}$' can then be compared with the desired beam pattern '$\underline{Z}$', as well as with a conventional beam pattern '$\underline{Z}'$' obtained using linear phase shifts across the array aperture selected to "steer" the array to the dominant arrival angle for the selected source depth.

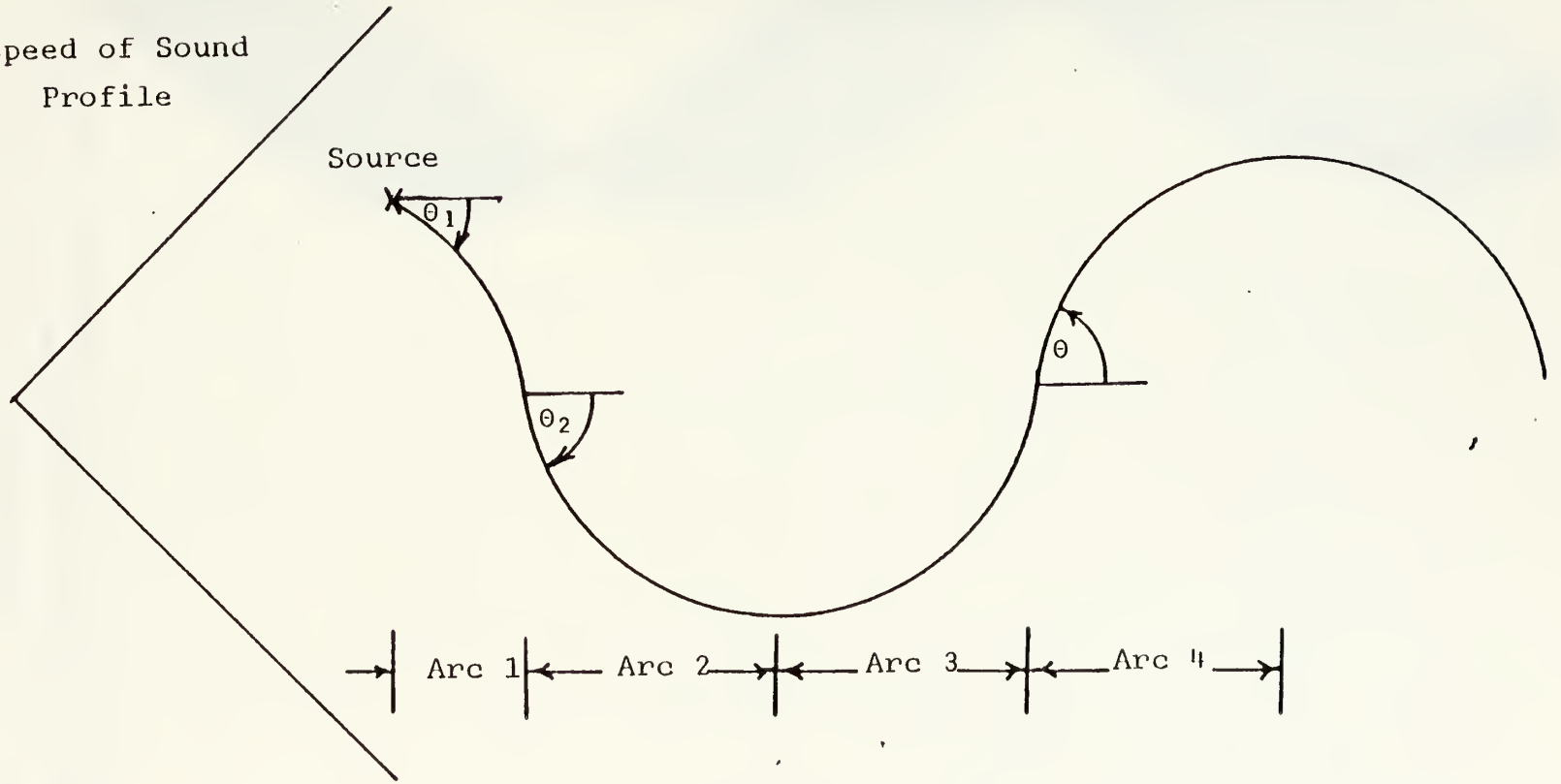

Figure 2.2 Single Ray Path Plot In Triangular SOFAR Channel

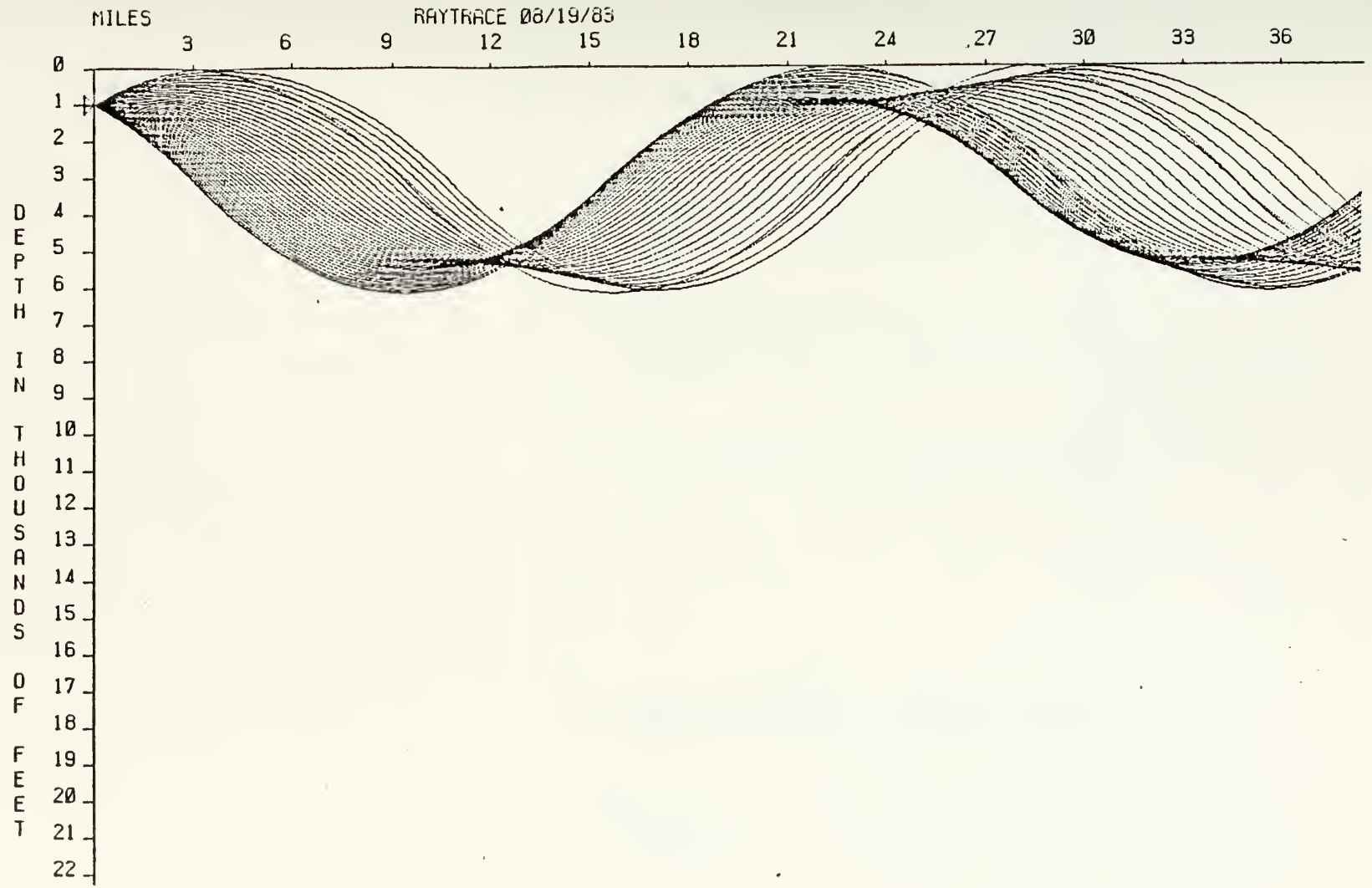

Figure 2.3 Ray Plot For Assumed Sound Channel

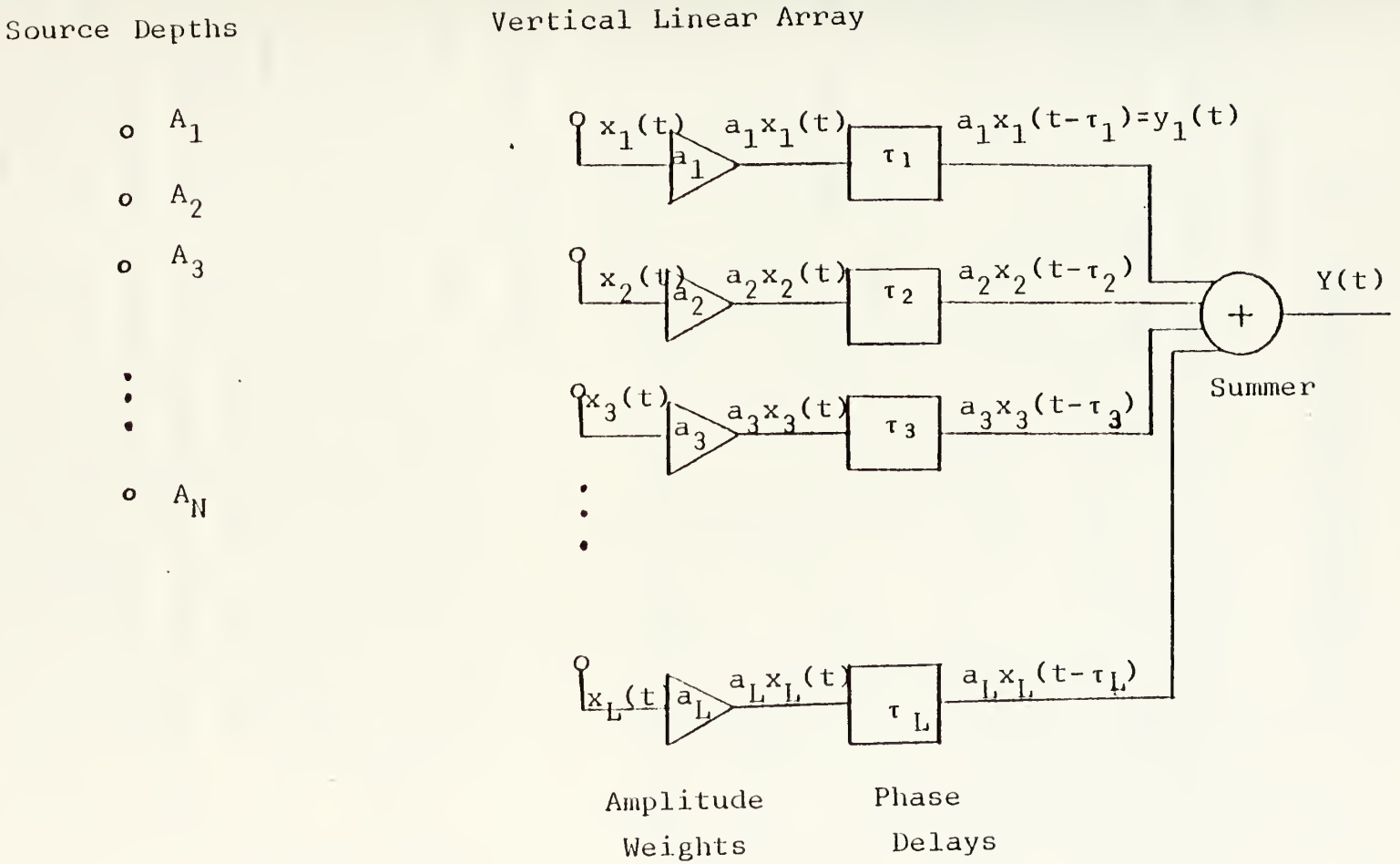

Figure 2.4 Array Model

III. EXPERIMENTAL PROCEDURE

A. BASIC ASSUMPTIONS

The speed of sound profile given in the Introduction was used to test the techinique, with 'L', the number of source depths chosen as 20. The source depths are at 220 meters and every 20 meters thereafter to 600 meters inclusive. 'N', the number of hydrophones used in the vertical array was chosen as 5. The hydrophones are placed at depths of 100, 200, 300, 400, and 500 meters. Thus, there are 40 equations, two for each source depth, and 10 unknowns, two for each hydrophone. A range of 200 kilometers (km) assures that the deep sound channel is filled with sound over the aperture of the vertical linear array. A frequency of 100 Hz provides good resolution in the beam pattern without introducing alias mainlobes at the selected range across the depths of investigation.

B. 'A' MATRIX CALCULATION

Since the gradient of the sound velocity, 'g' is constant in the area of the source depths, and the speed of sound at the surface is known, equation 2.3 is used to solve for 'c2', the speed of sound at the source depth. Equation 2.7 is used for each source depth to calculate the maximum initial depression angle which yields R-R rays. In determining travel time, only depression angles from each source which are positive (downward) and yield R-R rays are used.

Beginning with the first source depth of 220 meters, an initial depression angle of 0.0 degrees is selected. The ray path is then calculated using equations 2.5 and 2.6 and broken into a series of arcs as in figure 2.2. Equation 2.9

is used to determine the travel time for each arc in the same manner. Each arc's horizontal range, travel time, and depth are summed. When the summed horizontal range reaches 200 km the summation process ends. The travel time and depth of the ray at this horizontal range is then known. The same procedure is repeated for an initial depression angle of 0.1 degrees and every increment of 0.1 degrees thereafter until the maximum depression from equation 2.7 is reached. The final ray path is at this maximum depression angle.

The same procedure is repeated for each of the other 19 source depths.

Thus, for each initial angle from each source depth there is a ray which has a travel time and a depth when it reaches the horizontal range of 200 km. Since it is the profile of the sound pressure wave which impinges on the vertical array which is of importance, a constant can be subtracted from these calculated travel times. This constant is selected to be the travel time for the source depth of 220 meters which has an initial depression angle of 0 degrees. It is subtracted from each of the travel times making the resultant travel times relative with respect to the ray which has a 0 degree depression angle from the 220 meter depth. The program and its listing which calculates the relative travel times and the depths of these rays at the horizontal range of 200 km is given in Appendix A.

A plot of the relative travel times versus depth for the 220 meter source depth is shown in Figure 3.1. Figure 3.2 displays the plot for the 380 meter source depth. Negative relative travel times in the plots indicate that the overall travel time is less than the reference. These rays arrive at the 200 km horizontal distance before the reference ray.

Since the receiving hydrophones are at set vertical positions (100, 200, 300, 400, and 500 meters), an

interpolation is done to determine relative travel times to them from each source depth. The interpolation program and its listing is in Appendix B. Sometimes more than one R-R ray travels from the source depth to a hydrophone. When this occurs, the ray which arrives first is used in the calculation of relative travel time to that hydrophone.

Equation 2.12 determines the phase shift relating to the relative travel times. The '$\underline{A}$' matrix is formed by taking the appropriate sine and cosine values as in equation 2.24. The '$\underline{A}$' matrix is '40 by 10'.

C. '$\underline{Z}$' MATRIX

Referring to equation 2.26, the '$\underline{Z}$' matrix is a '40 by 1' column vector. It is the desired beam pattern. The bottom 20 rows give the imaginary terms and are set to zero. The top 20 rows represent the value of the real terms at each source depth. Therefore each of the top 20 rows is set to zero except for the row containing the source. It is set to 1. For example, if the source is at 220 meters then only the top row is set to 1. If the source is at 380 meters then only the ninth row is set to 1.

D. RESULTING BEAM PATTERN

1. Using the Linear Minimum Variance Method

'$\hat{\underline{e}}_{LMV}$' is calculated using equation 2.47. The resulting beam pattern '$\hat{\underline{Z}}$' is calculated using equation 2.51. The program which calculates the '$\underline{A}$' matrix, uses it in determining '$\hat{\underline{\theta}}_{LMV}$', and then calculates '$\hat{\underline{Z}}$' is given in Appendix C. The program listing is also included.

2. Using Linear Phase Shifts

The conventional beam pattern is determined by using equation 2.51 where '$\hat{\underline{\theta}}$' is calculated by approximating the plot of relative travel time vs. depth by a straight line at the receiving hydrophone depths. For example Figure 3.3 represents this plot for the 380 meter source depth. The straight line is determined by a least squares linear regression which minimizes the sum of the squares of the deviations of the actual data points from the straight line of best fit. Note that only data points which are on the dominant curve are used in calculating the straight line. From the straight line, relative travel times to the receiving hydrophones are calculated. The relative travel times for the 380 meter source depth are given in Table I. They correspond to a plane wave arrival angle of 3.73 degrees. '$\hat{\underline{\theta}}$' is determined by converting these relative travel times to phase delays using equation 2.12 and then taking the appropriate sine and cosine values of these phase delays as in equation 2.24. The amplitude weights are initially assumed to be unity.

A second method for obtaining the conventional beam pattern is calculated by the same procedure except the amplitude weights which are determined by equation 2.50, the '$\hat{\underline{\theta}}_{LMV}$' amplitude amplitude weights, are applied to each hydrophone.

TABLE I

Relative Travel Times For 380 Meter Source Depth

Hydrophone Depth	Relative Travel Time
100 meters	-0.07565509 sec.
200 meters	-0.07131599 sec.
300 meters	-0.06697690 sec.
400 meters	-0.06263780 sec.
500 meters	-0.05829871 sec.

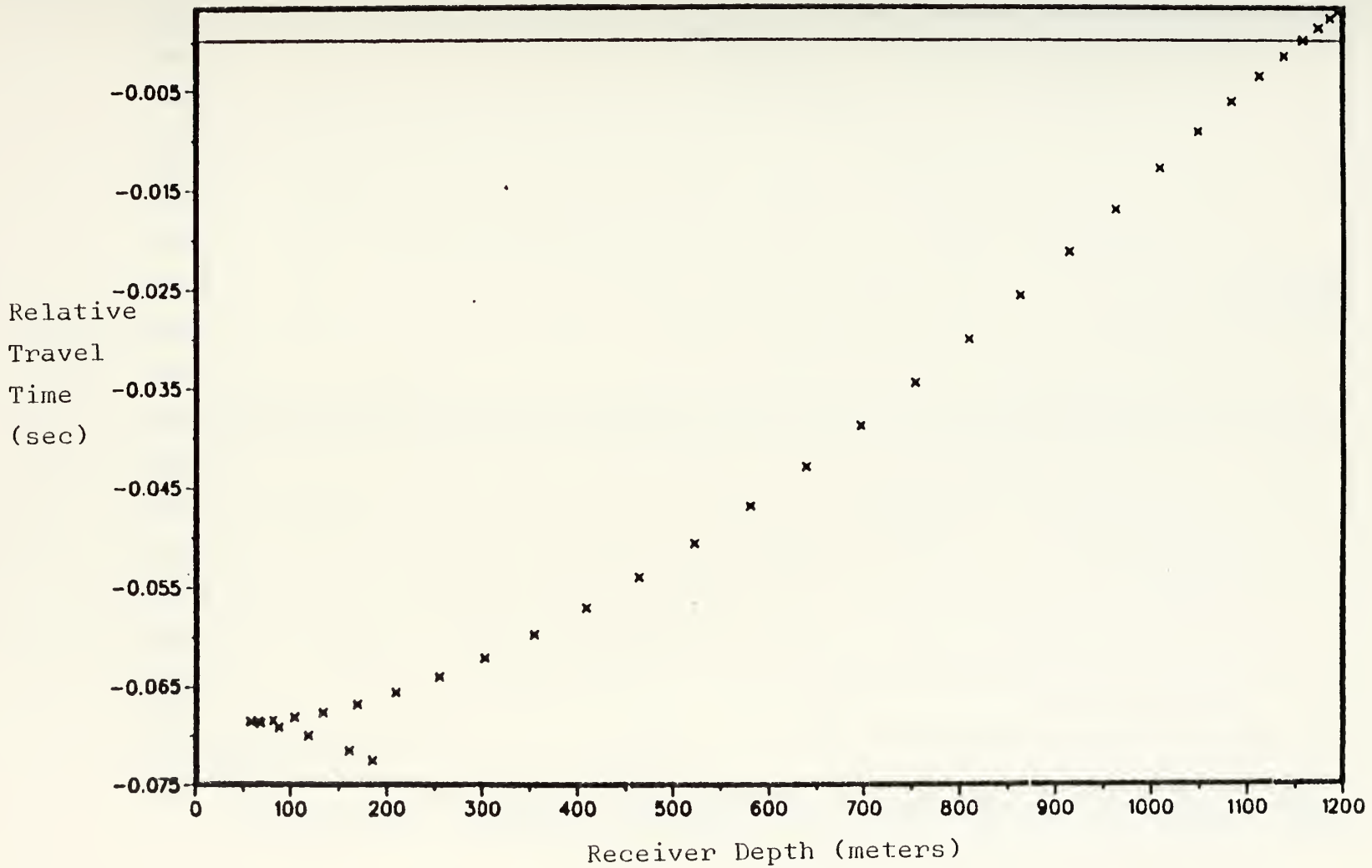

Figure 3.1 Relative Travel Time vs. Depth (220 meter source)

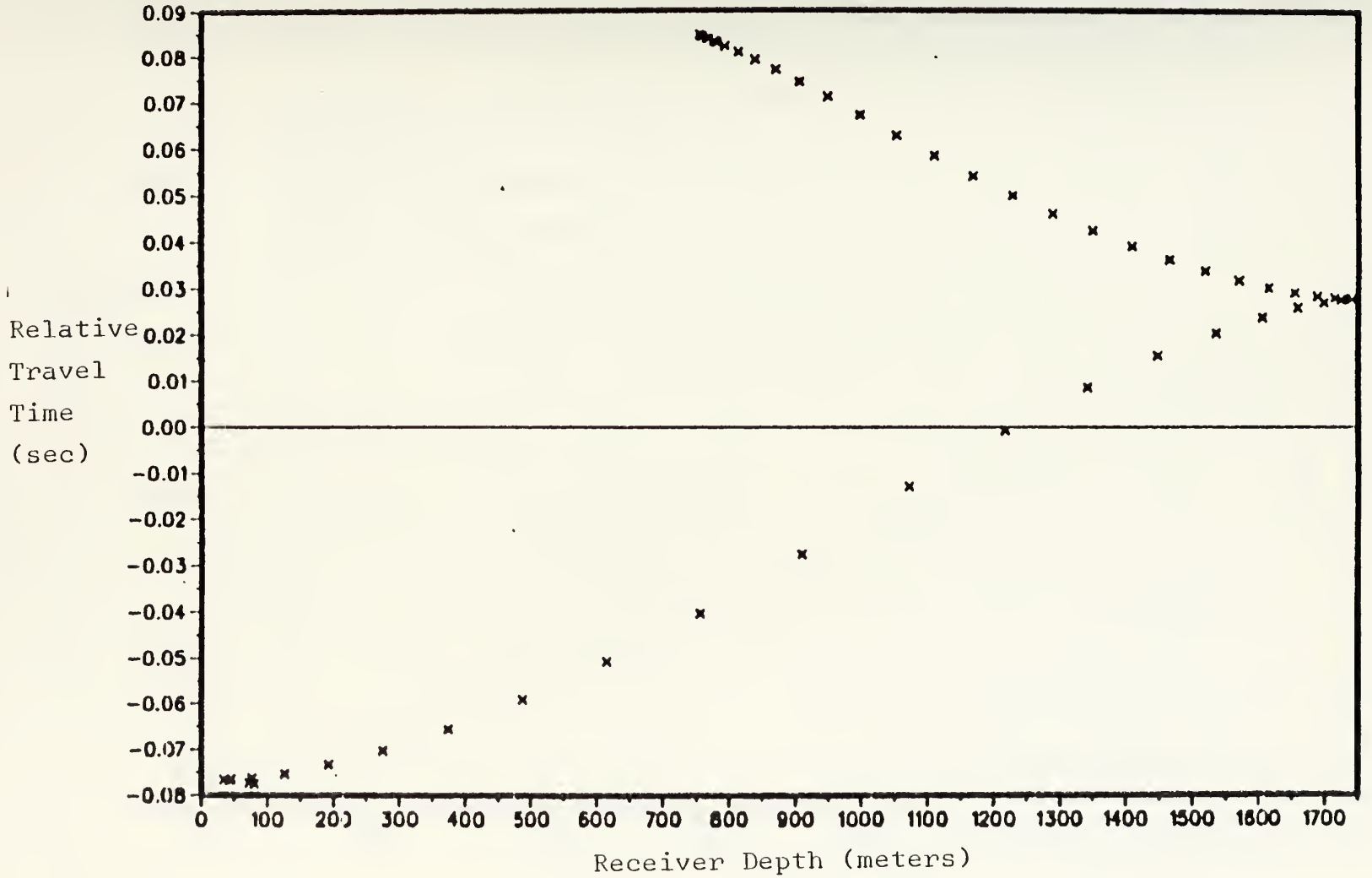

Figure 3.2 Relative Travel Time vs. Depth (380 meter source)

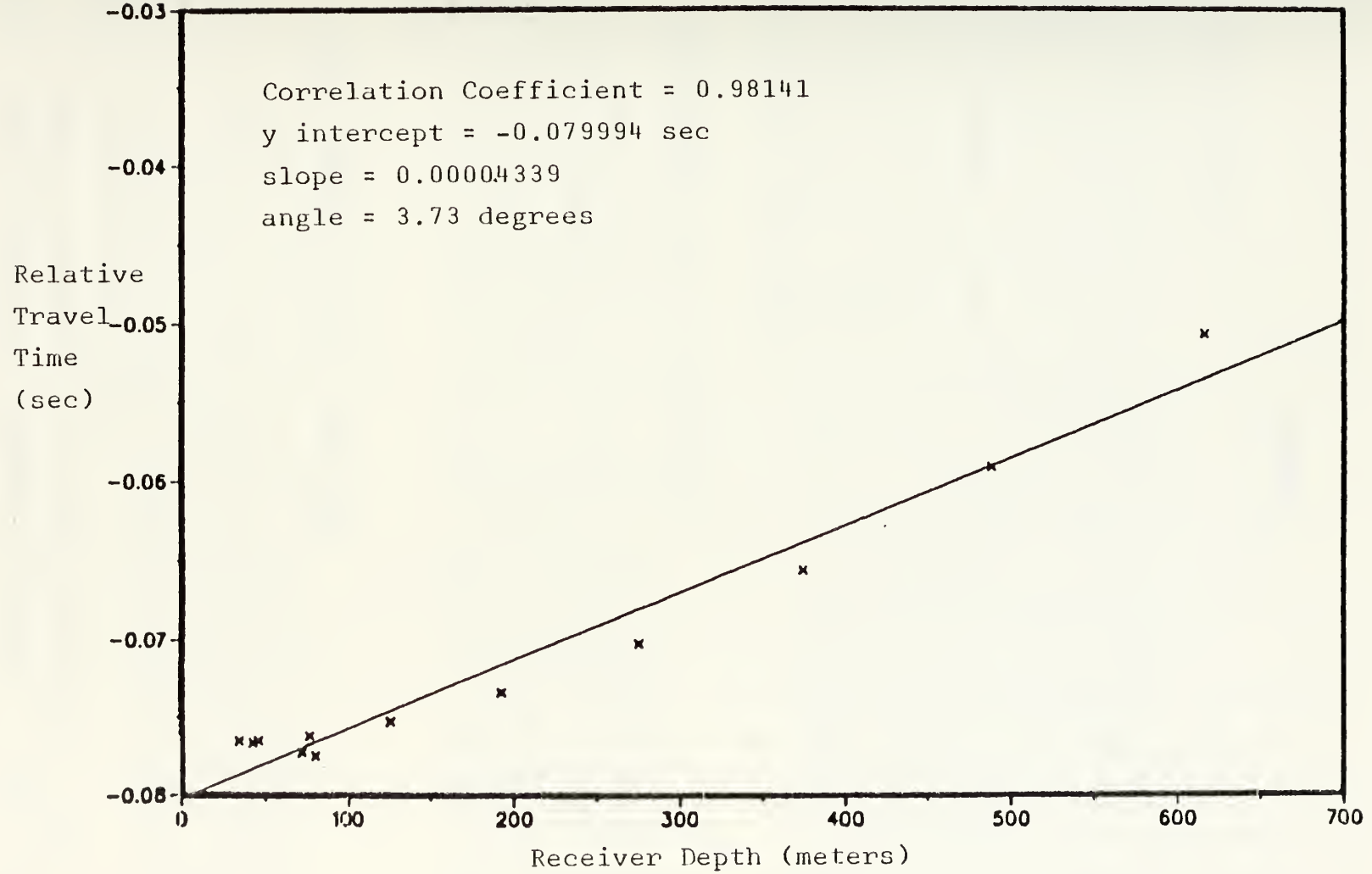

Figure 3.3 Straight Line Approx. of Rel. Trav. Time vs. Depth (source at 380 m)

IV. RESULTS

A. EXACT SOLUTION

An exact solution is derived for the beam pattern if the number of receiving hydrophones equal the number of source depths. For example, when the 5 receivers are used to discriminate between 5 source depths (220, 240, 260, 280, and 300 meters) there are 10 equations with 10 unknowns. Figure 4.1 is a plot of the resulting beam pattern with the source at the shallowest depth. Note that because of round-doff errors in the 'IMSL' subroutines there is a small value for the resulting beam pattern at the non-energy source depths under investigation.

B. FOUR DEPTHS WITH TWO RECEIVERS

For source depths at 220, 280, 300, and 320 meters with receiving hydrophones at 100 and 200 meters, there are 8 equations with 4 unknowns. Figure 4.2 is a plot of the resulting beam pattern with a source at the 220 meter depth. Figure 4.3 is the plot for the source at the 280 meter depth.

C. TWENTY DEPTHS WITH TWO RECEIVERS

For all 20 source depths with receiving hydrophones at 100 and 200 meters, there are 40 equations with 4 unknowns. Figure 4.4 is a plot of the resulting beam pattern for a source at the 220 meter depth. Figure 4.5 is a plot for the source at the 380 meter depth.

In order to determine if the '$\underline{\hat{\theta}}$' calculated in this case is the best an alternate method is devised. Four source

depths (220, 240, 260 meters, and another source test depth) with the original source at the 220 meter depth and receivers at 100 and 200 meters are used. The beam pattern is calculated each time with a different source test depth substituted for the fourth source depth. The 5 best resulting beam patterns are selected along with the 8 source depths (220, 240, 260 meters, and the 5 test depths which created the 5 best beam patterns). Then, using these 8 source depths, '$\hat{\underline{\theta}}$' is determined for the two receivers. This '$\hat{\underline{\theta}}$' is applied to the two receivers and the beam pattern obtained for all 20 source depths.

The resulting beam pattern obtained by this alternate method isn't as good as the beam pattern obtained by using all 20 source depths in the determination of '$\hat{\underline{\theta}}$'.

D. TWENTY DEPTHS WITH FIVE RECEIVERS

For all 20 source depths with all 5 receiving hydrophones there are 40 equations with 10 unknowns. Figure 4.6 is a plot of the resulting beam pattern with the source at the 220 meter depth. Figures 4.7, 4.8, and 4.9 are the plots for the source at the 360, 380, and 400 meter depths respectively.

E. CONVENTIONAL BEAMFORMER

Figure 4.10 is a plot of the beam pattern for a conventional beamformer using linear phase shifts across the array with the source at the 380 meter depth and the amplitude weights set to unity. All 20 source depths and 5 receiving hydrophones are used. Note that in figure 4.10 that there is less than 1 db discrimation between each of the source depths. Figure 4.11 is the plot obtained for the amplitude weights set to values determined by equation 2.50.

F. RANGE OF 250 KILOMETERS

The calculations were repeated for a range of 250 km using the same 20 source depths and 5 receiving hydrophones. Figures 4.12 and 4.13 represent the plots of relative travel times versus depth for the 220 and 380 meter source depths respectively. Figure 4.14 represents the straight line approximation of the relative travel times for the 380 meter depth. Note that in this figure the relative travel times are represented by two straight lines; the upper line represents linear phase shifts of the slower travel times for the conventional beamformer while the lower line represents linear phase shifts of the faster travel times. Tables II and III are the straight line interpolations of these slower and faster travel times which correspond to arrival angles of 4.04 and -3.16 degrees respectively. Figures 4.15 and 4.16 are the resulting beam patterns for the conventional beamformer for the slower travel times using unity amplitude weights and linear minimum variance amplitude weights respectively. Figures 4.17 and 4.18 are the beam patterns for the faster travel times.

Figures 4.19, 4.20, 4.21, and 4.22 represent plots of the beam pattern for the source at the 220, 340, 360, and 380 meter depths respectively.

TABLE II

Slower Ray Travel Times For 250 km Range and 380 m Source

Hydrophone Depth	Relative Travel Time
100 meters	-0.09949109 sec.
200 meters	-0.09478615 sec.
300 meters	-0.09008121 sec.
400 meters	-0.08537627 sec.
500 meters	-0.08067133 sec.

TABLE III

Faster Travel Times For 250 km Range and 380 m Source

Hydrophone Depth	Relative Travel Time
100 meters	-0.10100745 sec.
200 meters	-0.10468763 sec.
300 meters	-0.10836781 sec.
400 meters	-0.11204799 sec.
500 meters	-0.11572817 sec.

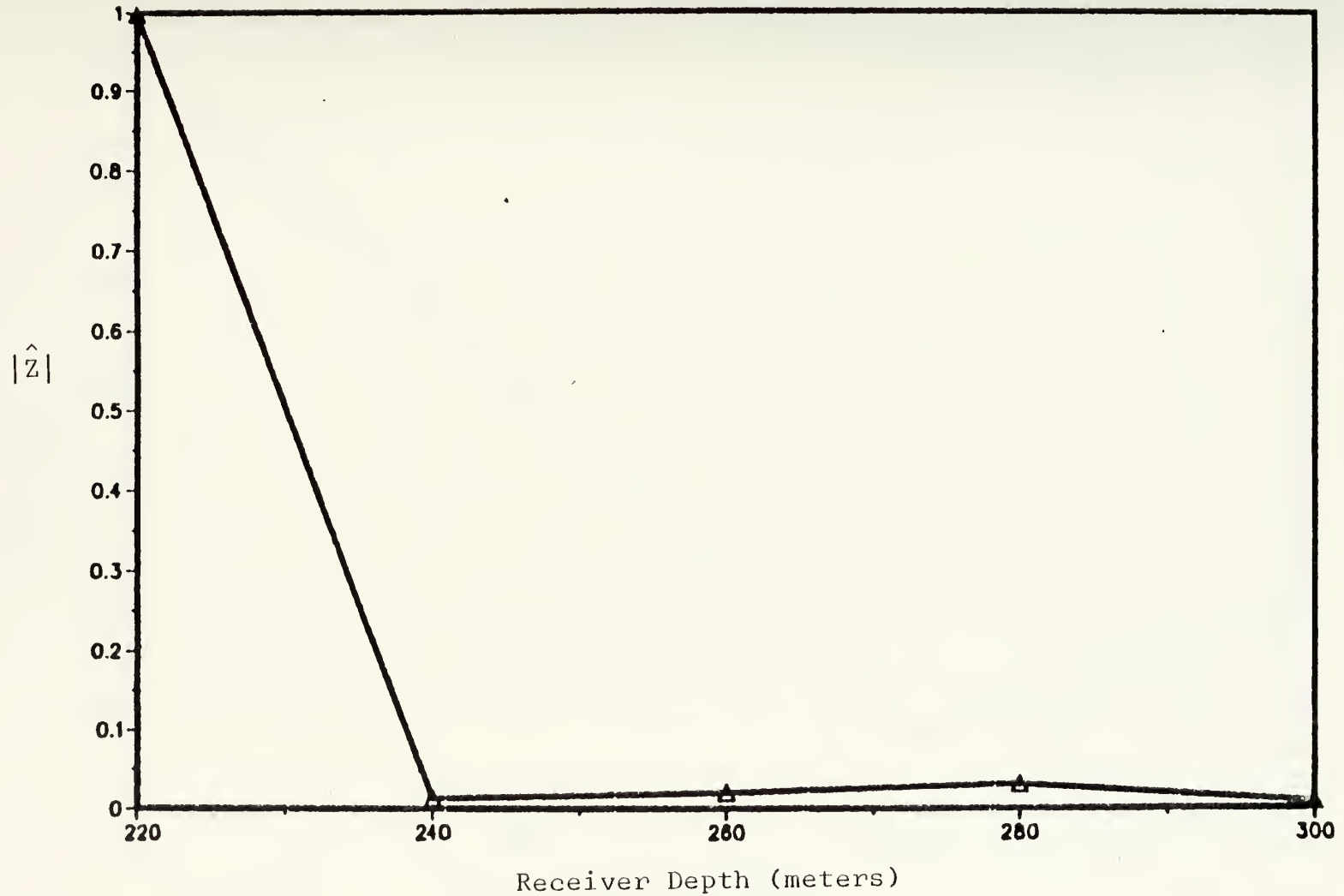

Figure 4.1 Beam Amplitude vs. Source Depth (5 depths 5 receivers)

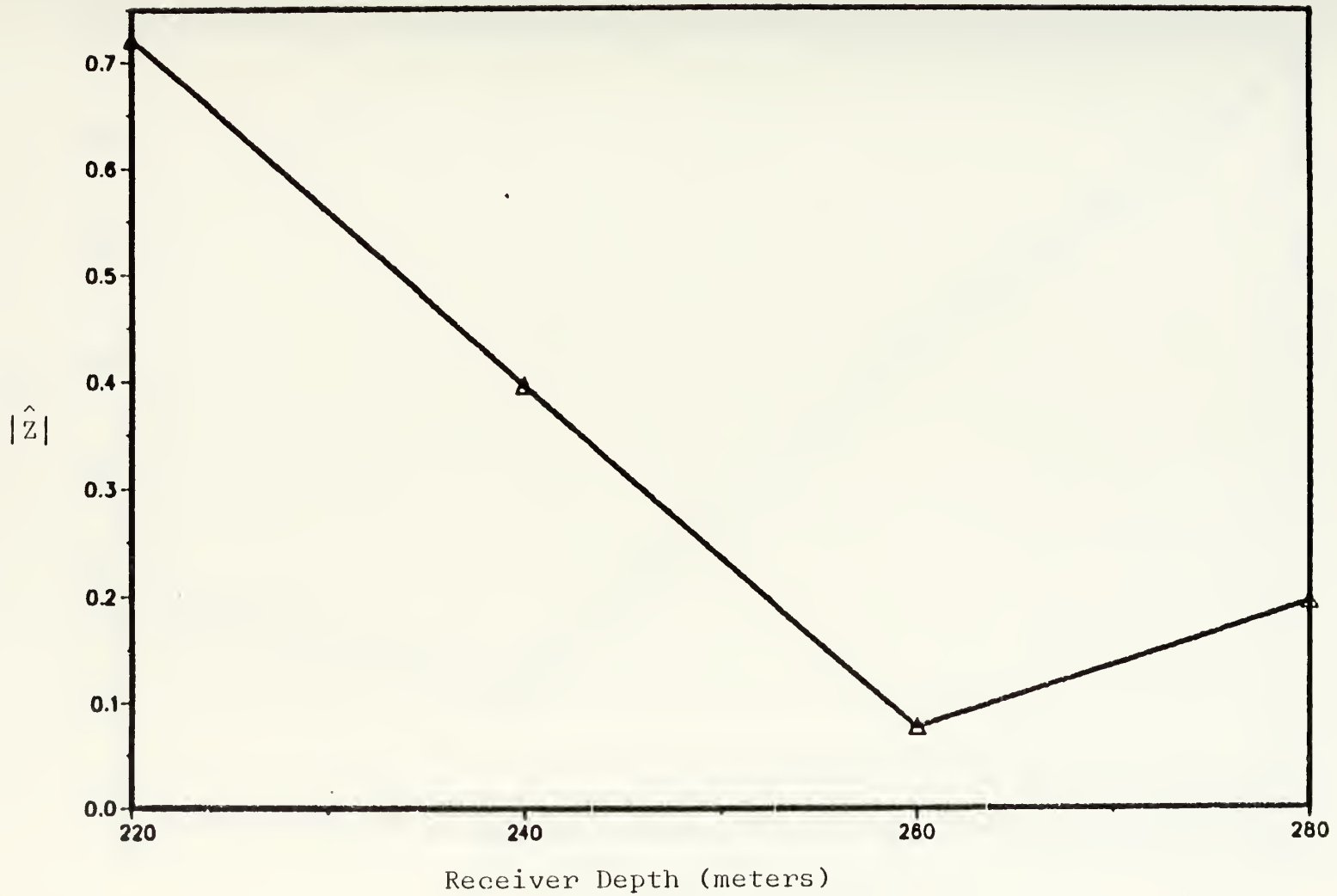

Figure 4.2 Beam Pattern (4 depths 2 receivers, source at 220 m)

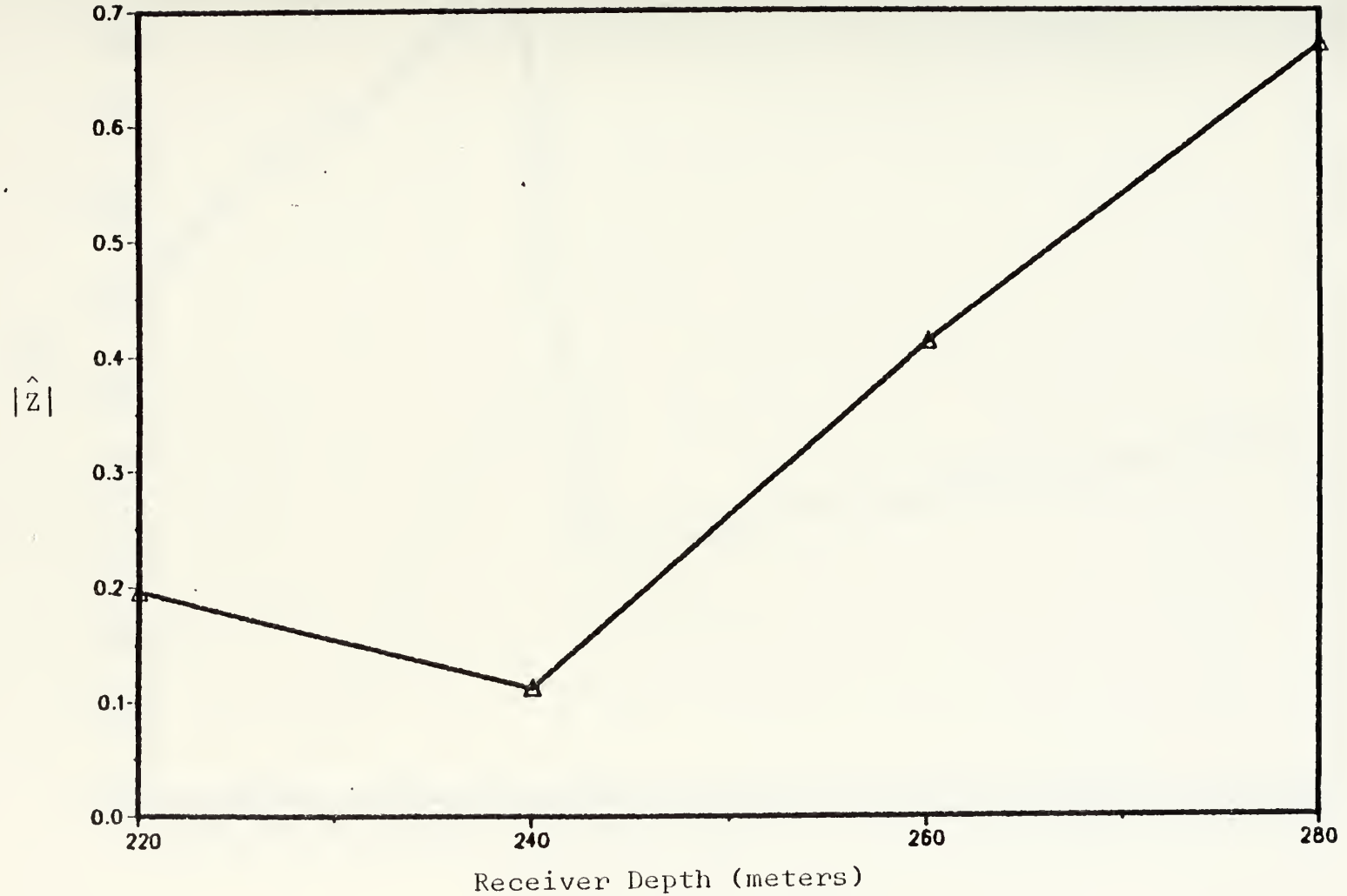

Figure 4.3 Beam Pattern (4 depths 2 receivers, source at 280 m)

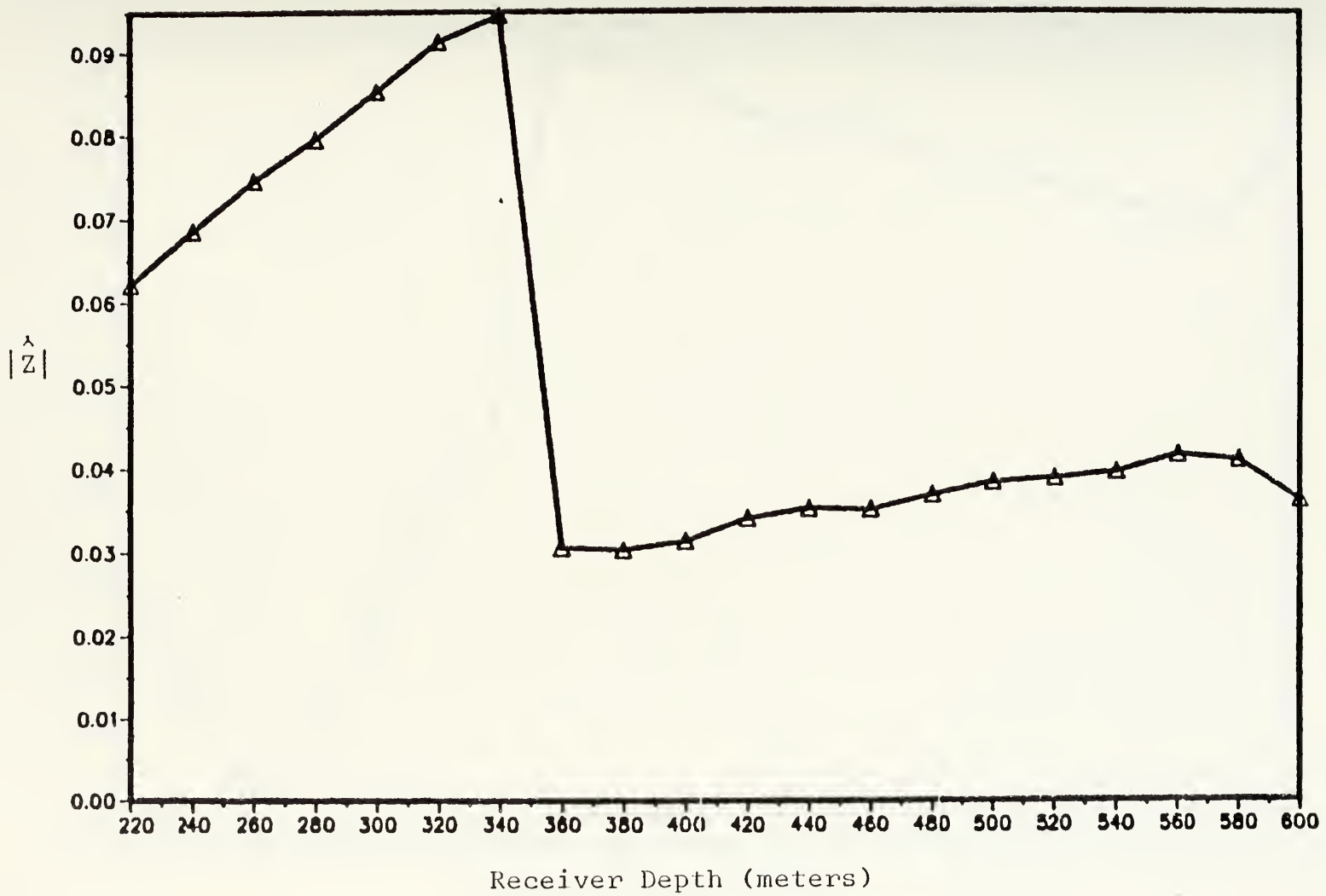

Figure 4.4 Beam Pattern (20 depths 2 receivers, source at 220 m)

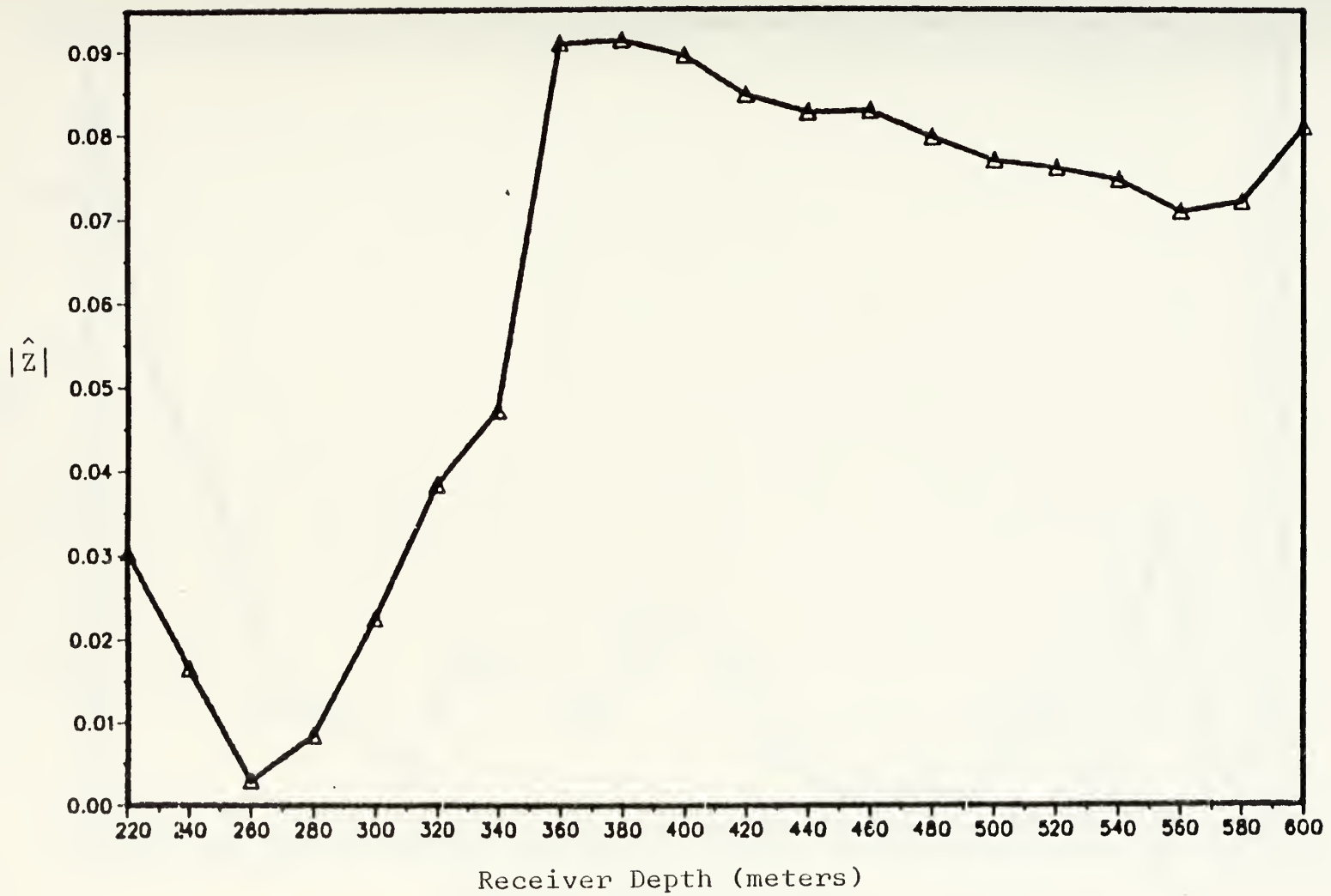

Figure 4.5 Beam Pattern (20 depths 2 receivers, source at 380 m)

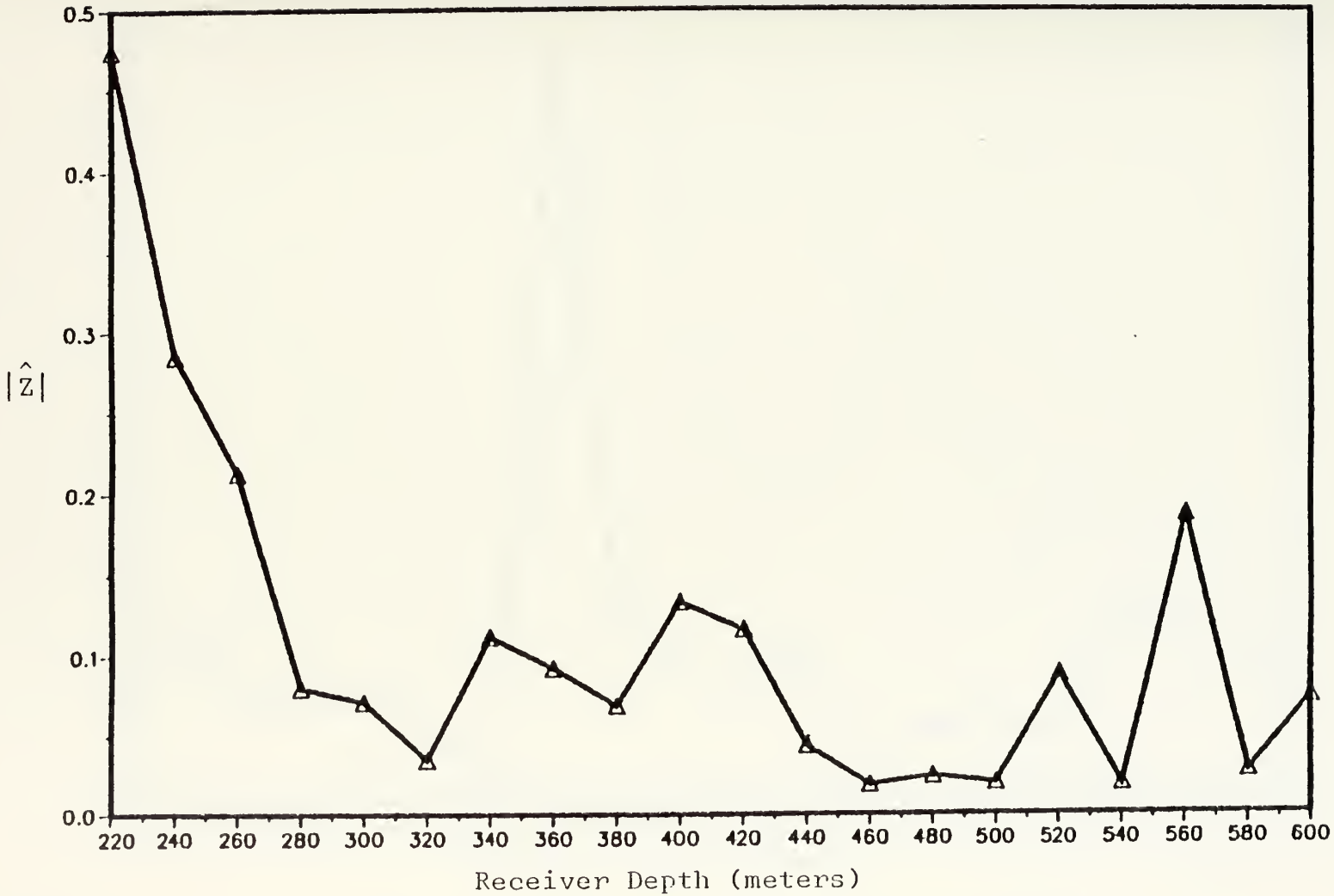

Figure 4.6 Beam Pattern (20 depths 5 receivers, source at 220 m)

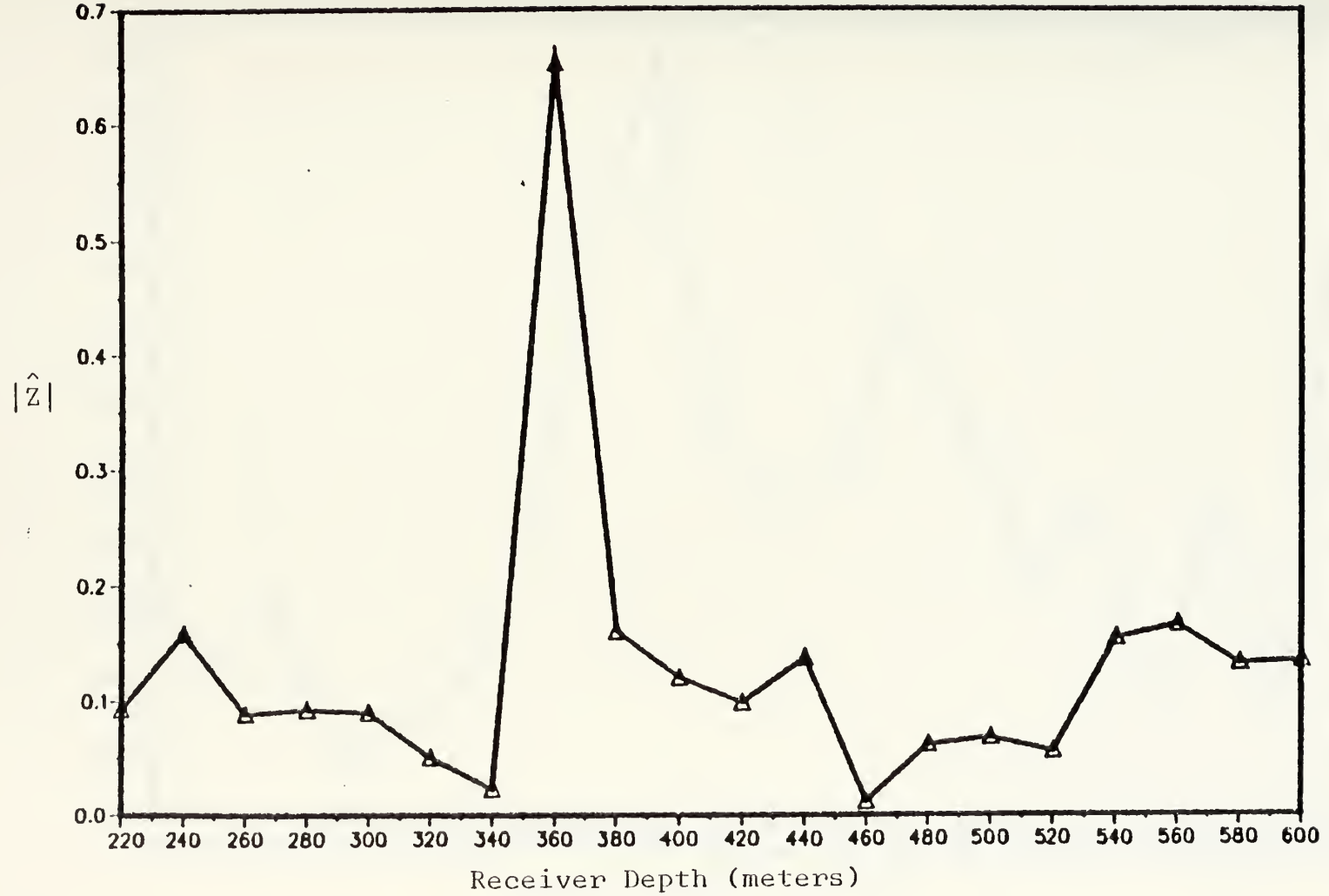

Figure 4.7 Beam Pattern (20 depths 5 receivers, source at 360 m)

Figure 4.8 Beam Pattern (20 depths 5 receivers, source at 380 m)

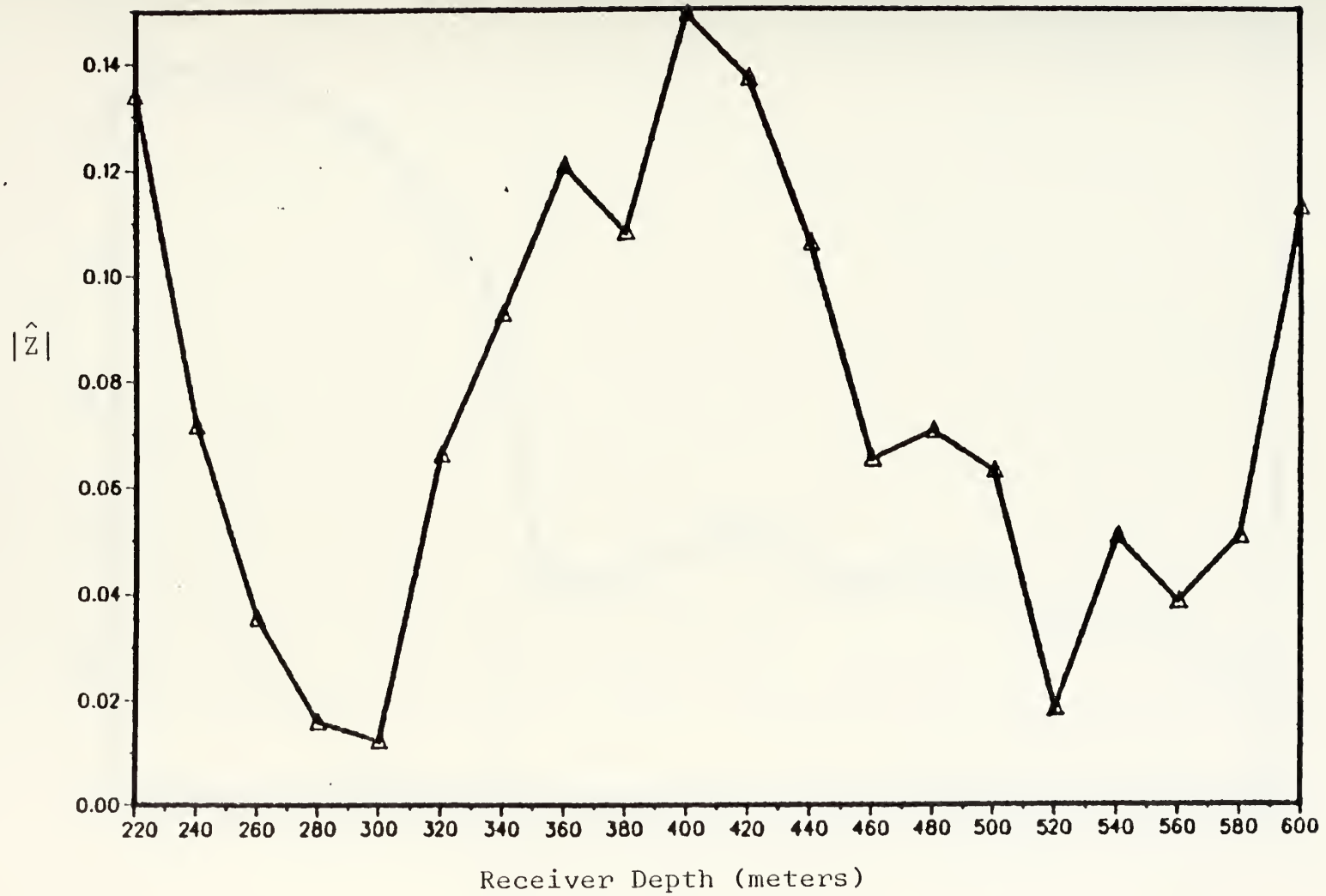

Figure 4.9 Beam Pattern (20 depths 5 receivers, source at 400 m)

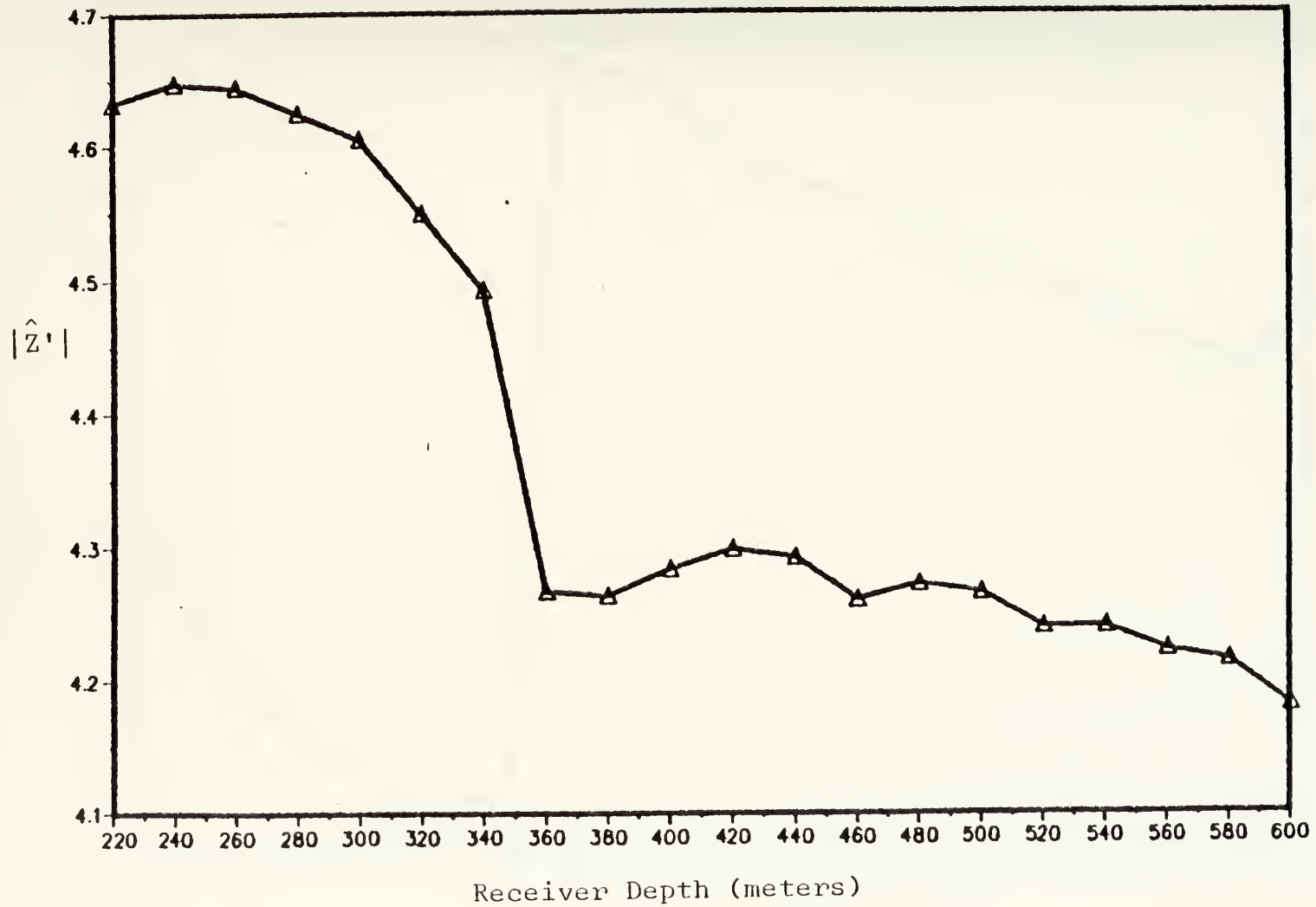

Figure 4.10 Conventional Beam Pattern (source at 380 meters)

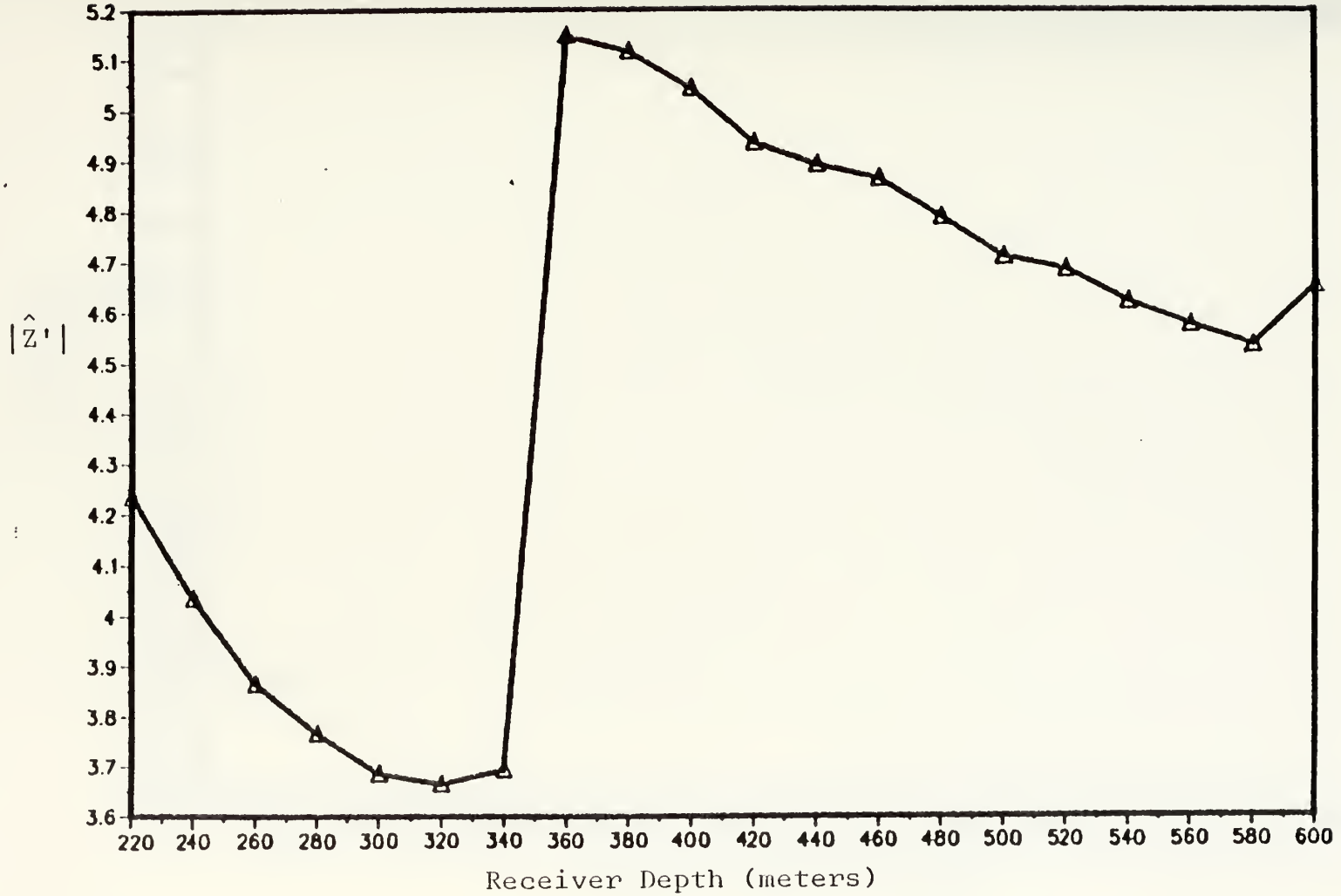

Figure 4.11 Conventional Beam Pattern (non-unity amp. wts. source at 380 m)

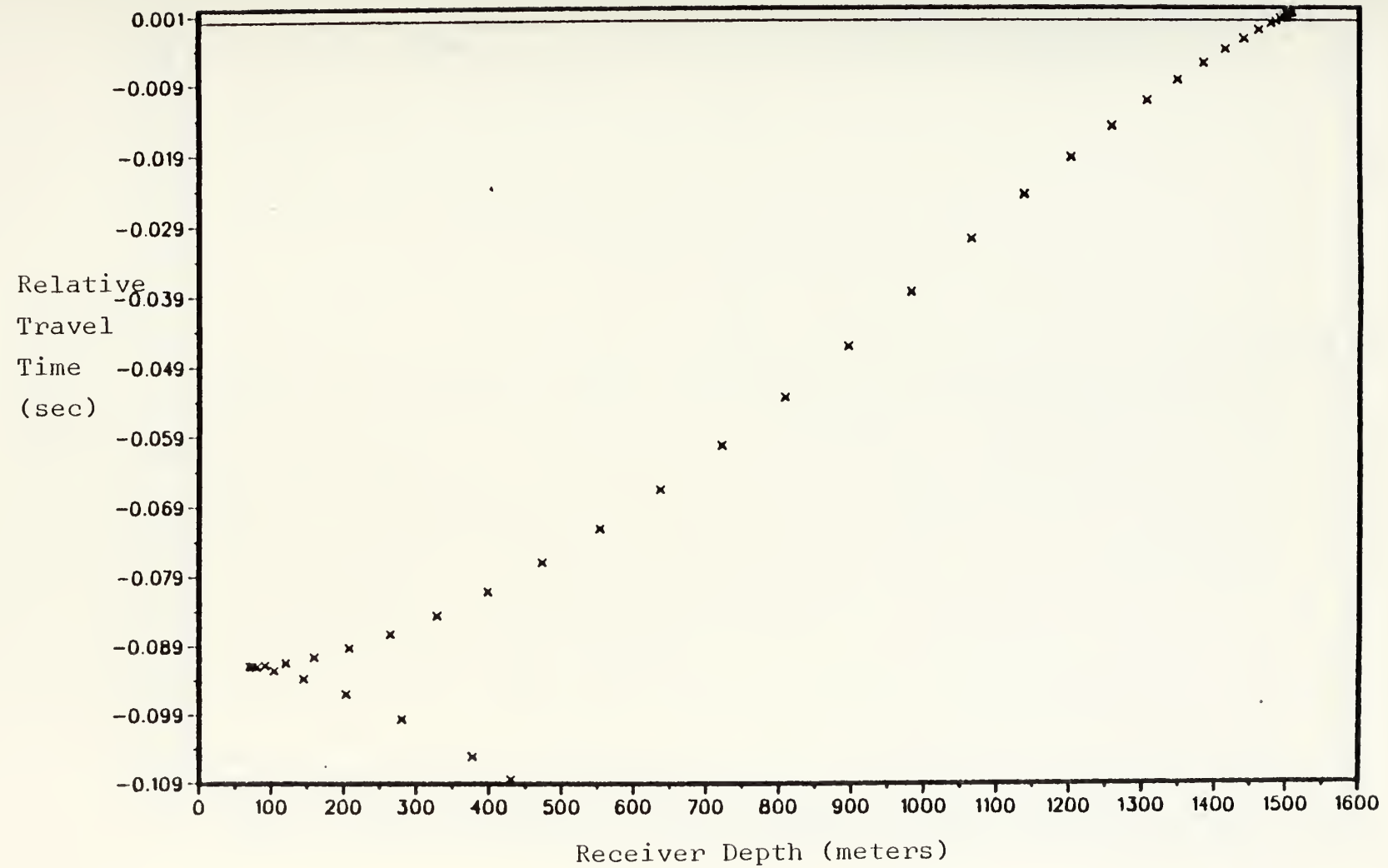

Figure 4.12 Relative Travel Time vs. Depth (range=250 km, source at 220 m)

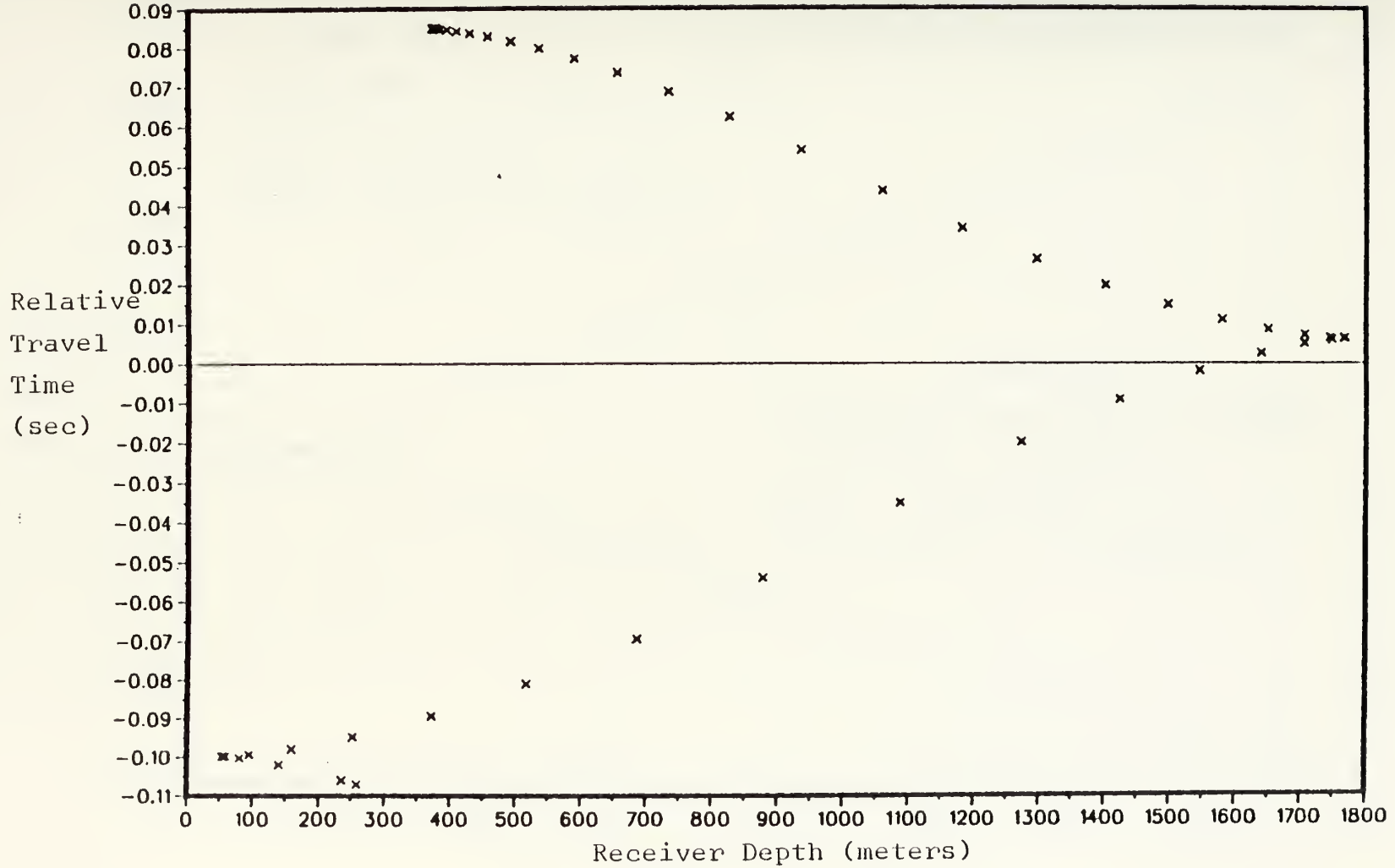

Figure 4.13 Relative Travel Time vs. Depth (range=250 km, source at 380 m)

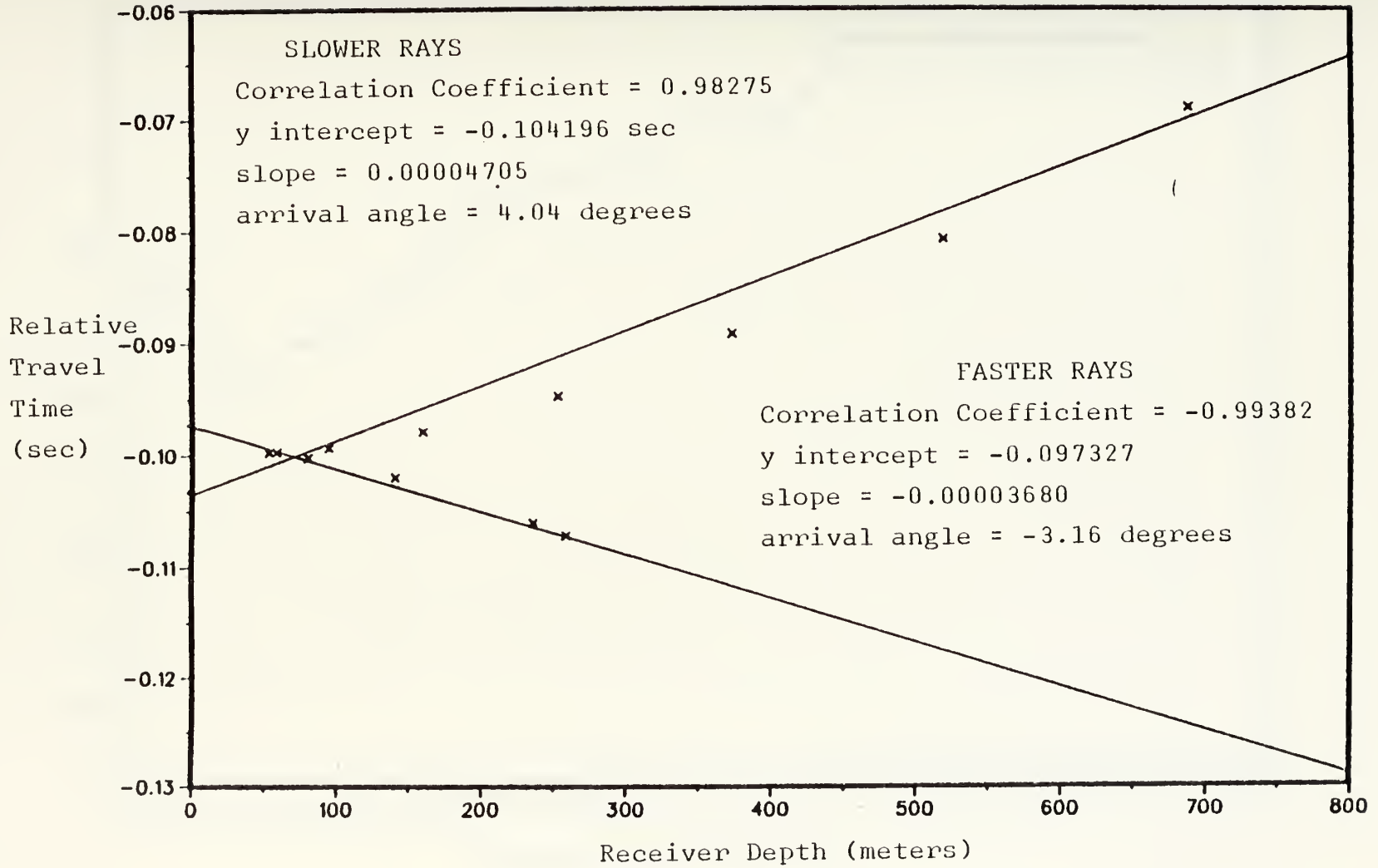

Figure 4.14 St. Line Approx. of Rel. Trav. Time vs. Depth (R=250 km, d=380 m)

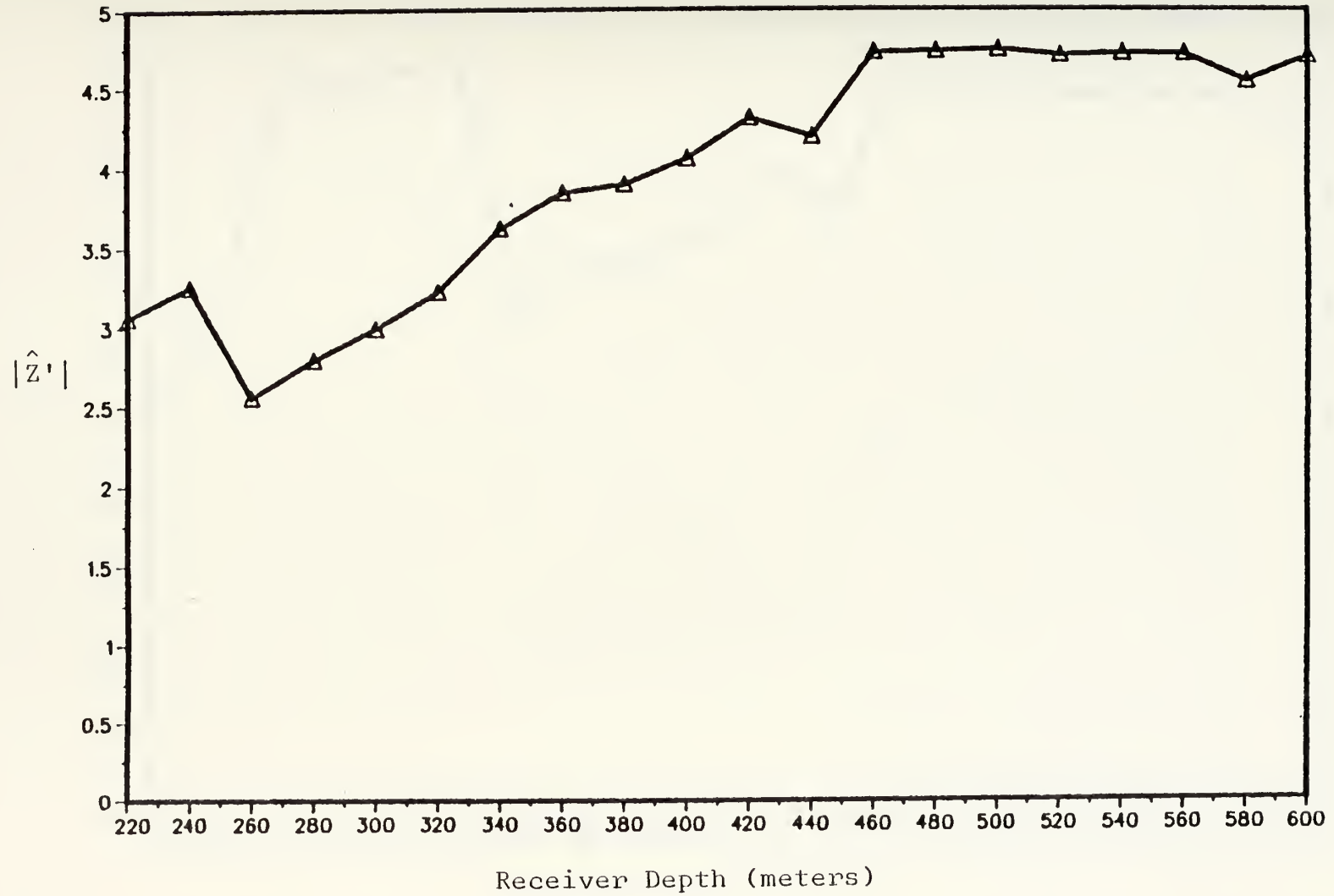

Figure 4.15 Conventional Beam Pattern (R=250 km, d=380 m, slower times, a=1)

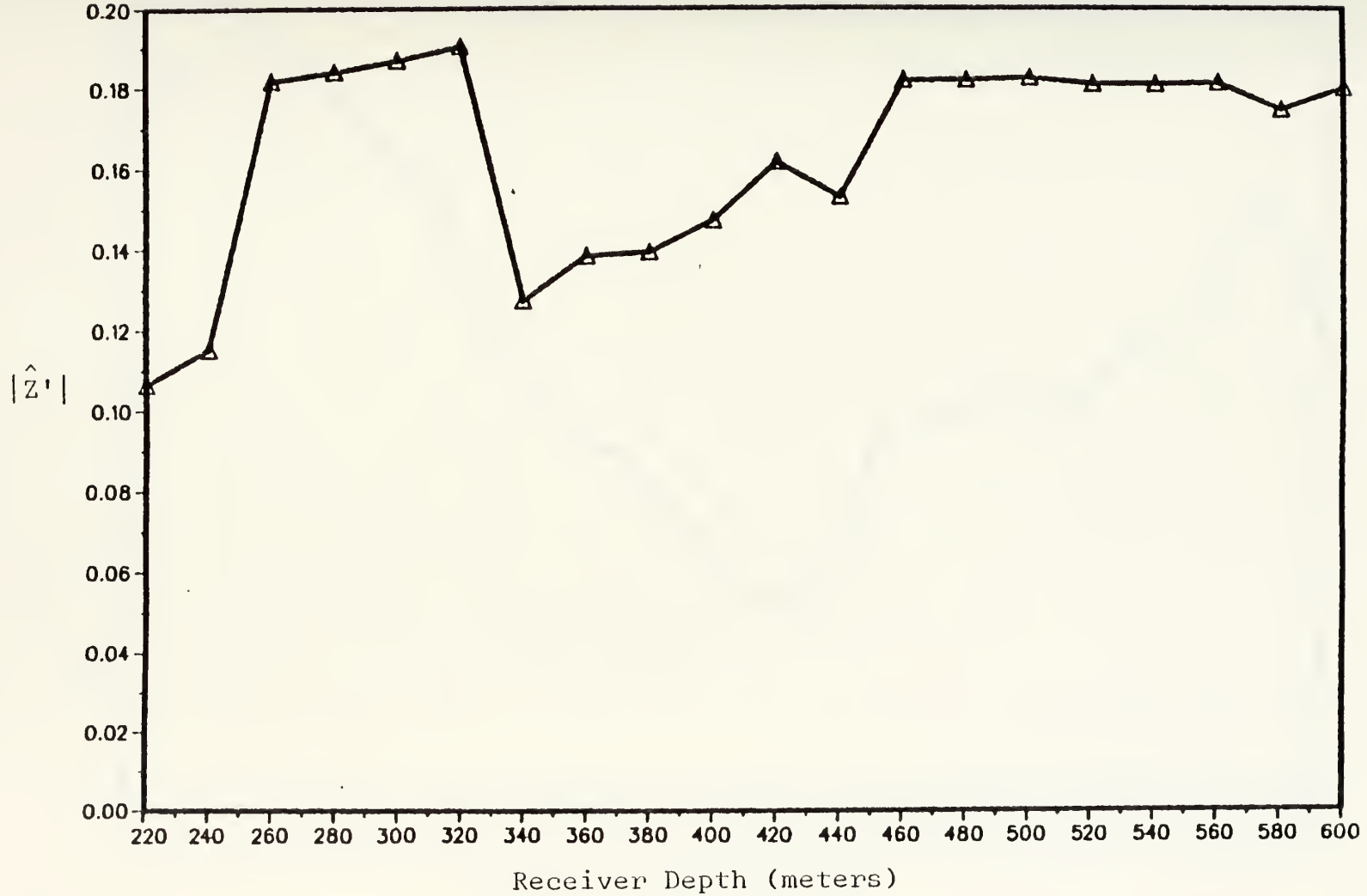

Figure 4.16 Conventional Beam Pattern (R=250 km, d=380 m, slower times, LMV a)

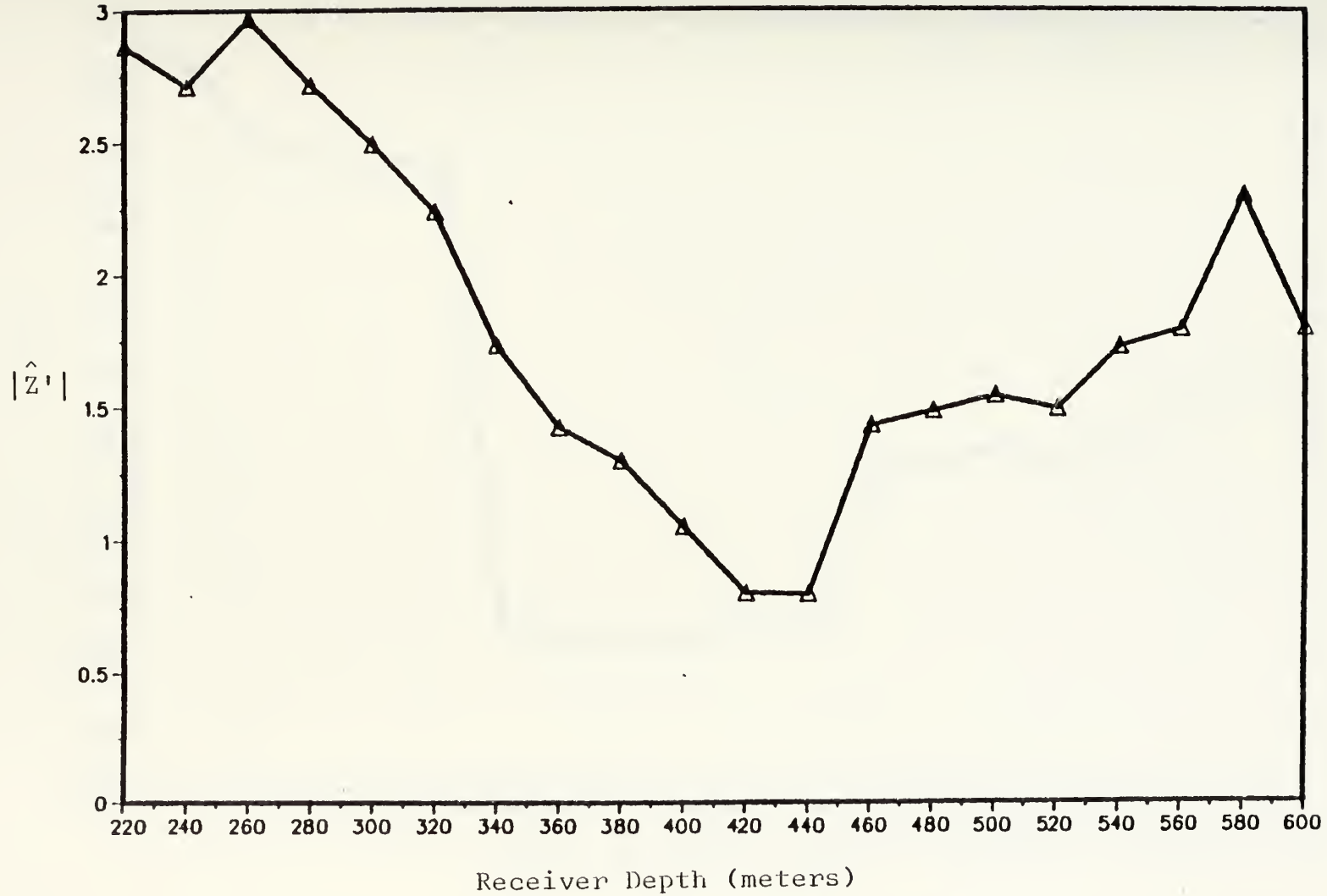

Figure 4.17 Conventional Beam Pattern (R=250 km, d=380 m, faster times, a=1)

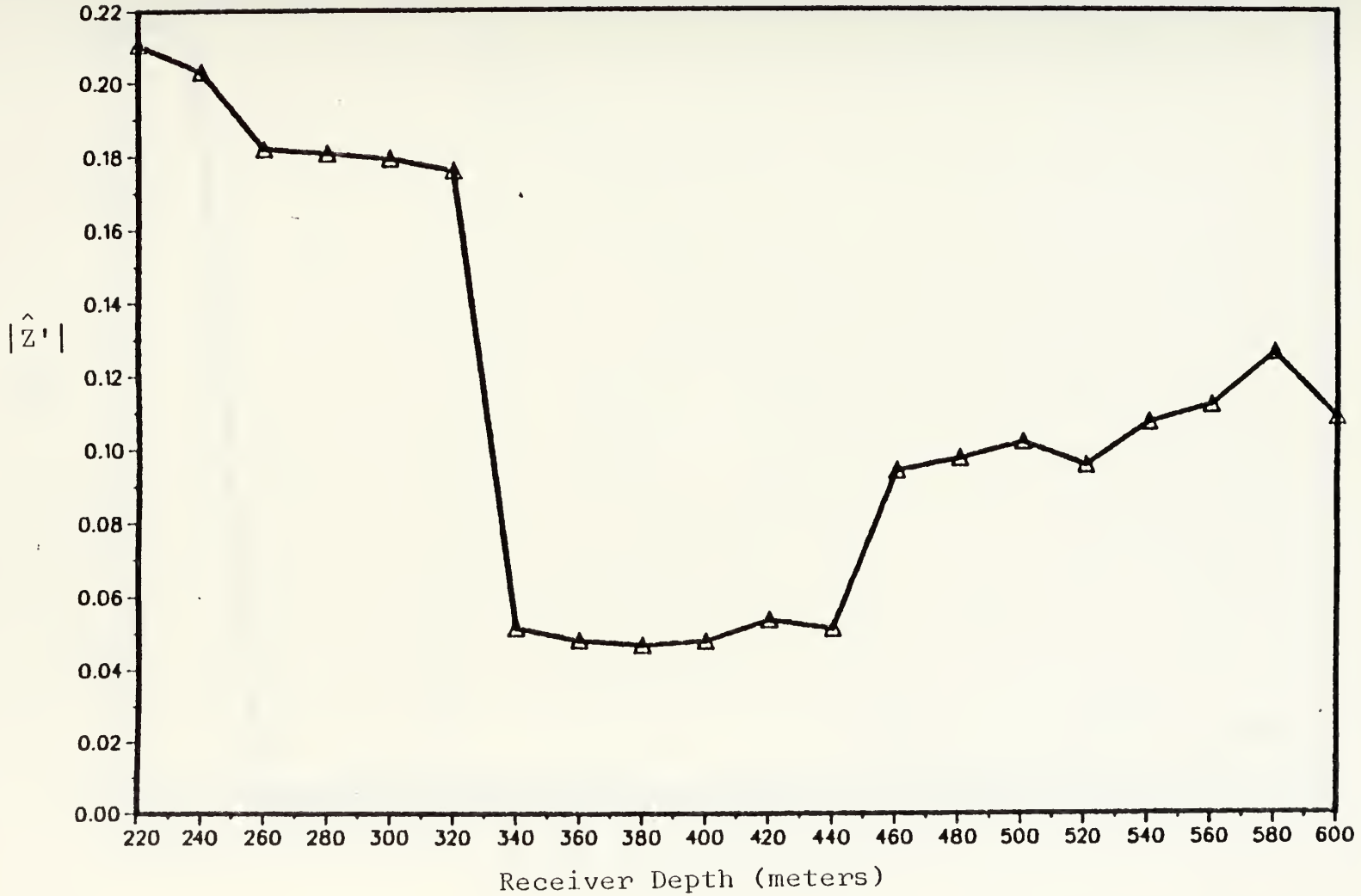

Figure 4.18 Conventional Beam Pattern (R=250 km, d=380 m, faster times, LMV a)

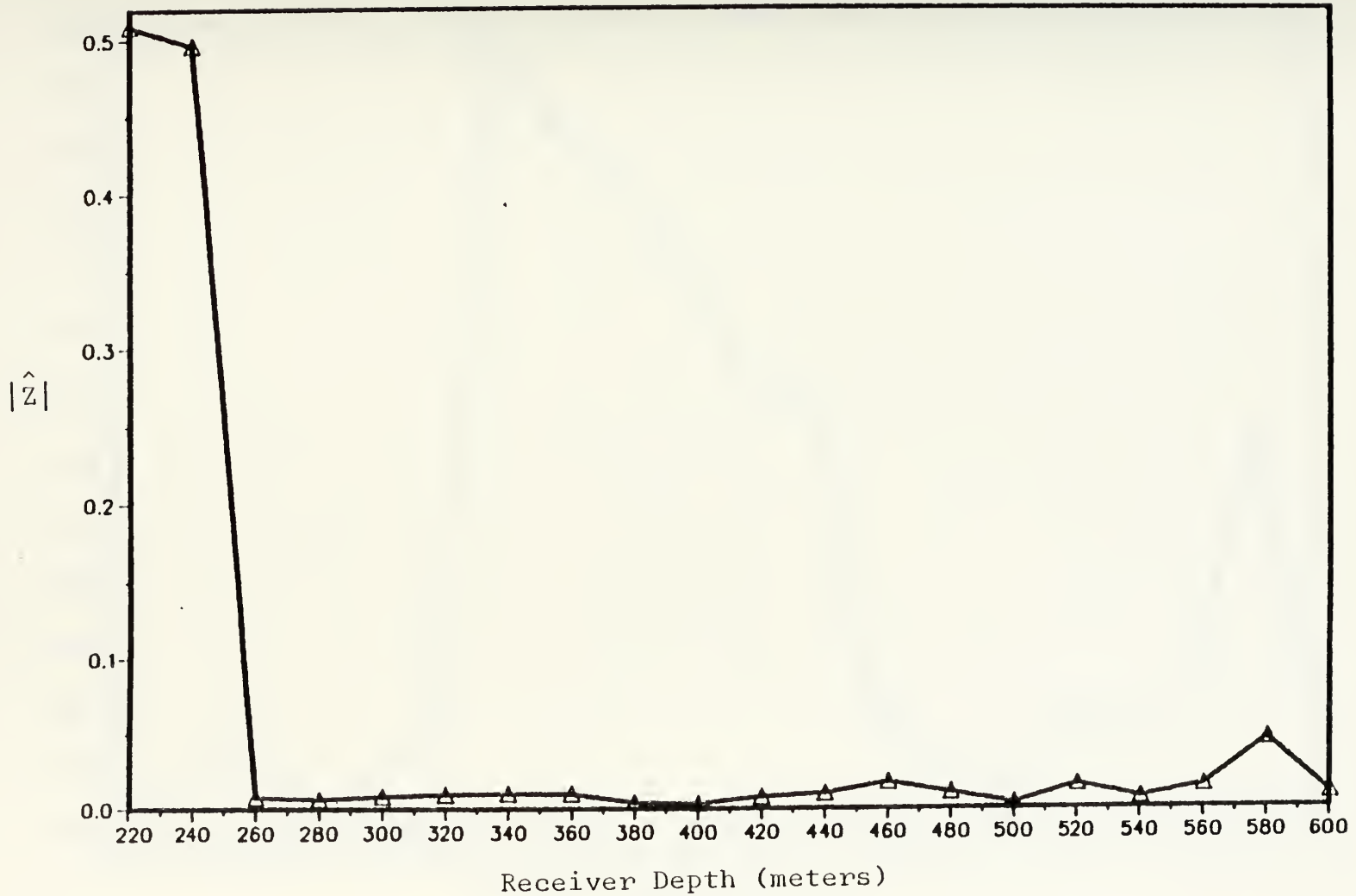

Figure 4.19 Beam Pattern (R=250 km, 20 depths, 5 receivers, source at 220 m)

Figure 4.20 Beam Pattern (R=250 km, 20 depths, 5 receivers, source at 340 m)

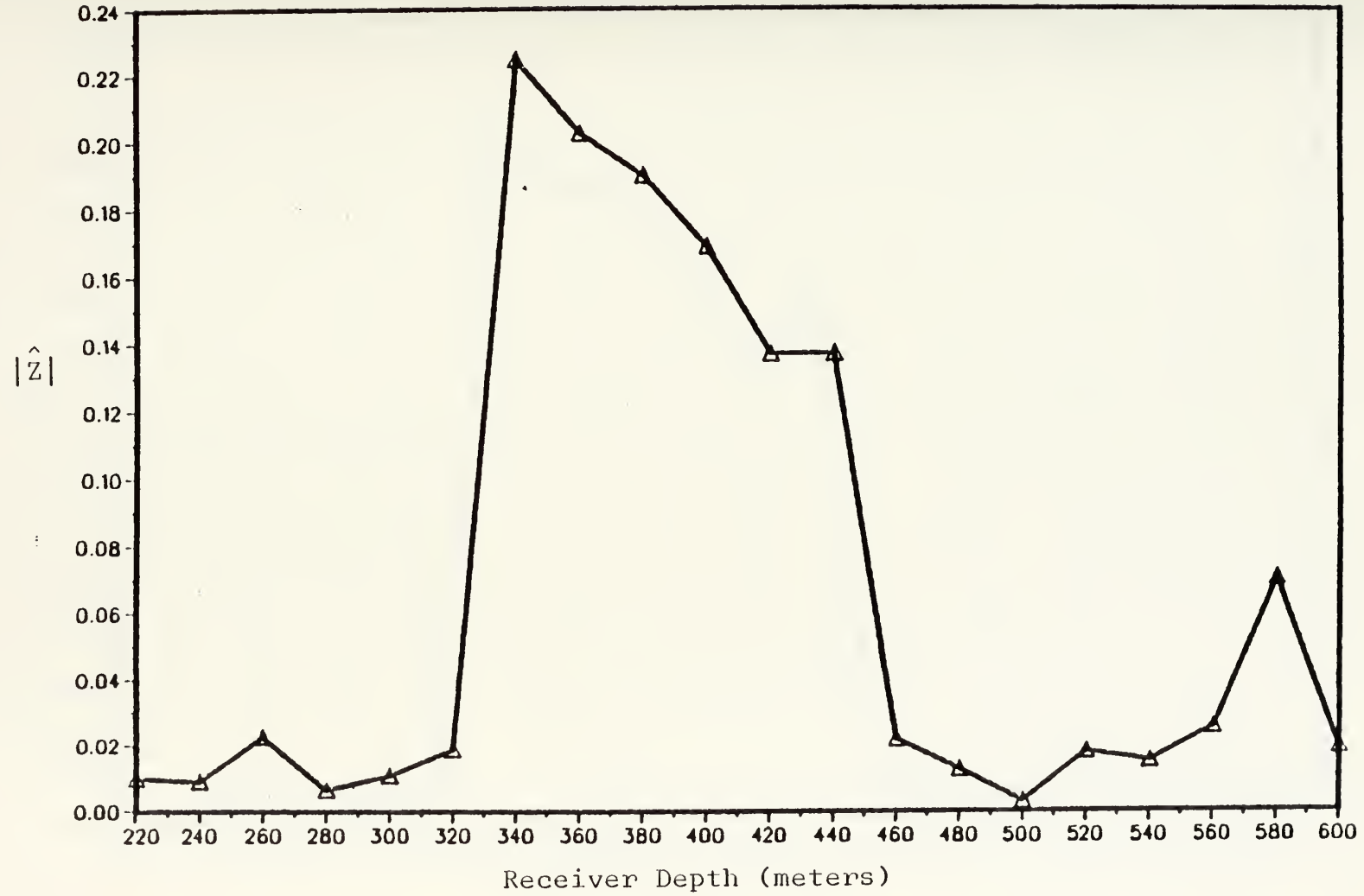

Figure 4.21 Beam Pattern (R=250 km, 20 depths, 5 receivers, source at 360 m)

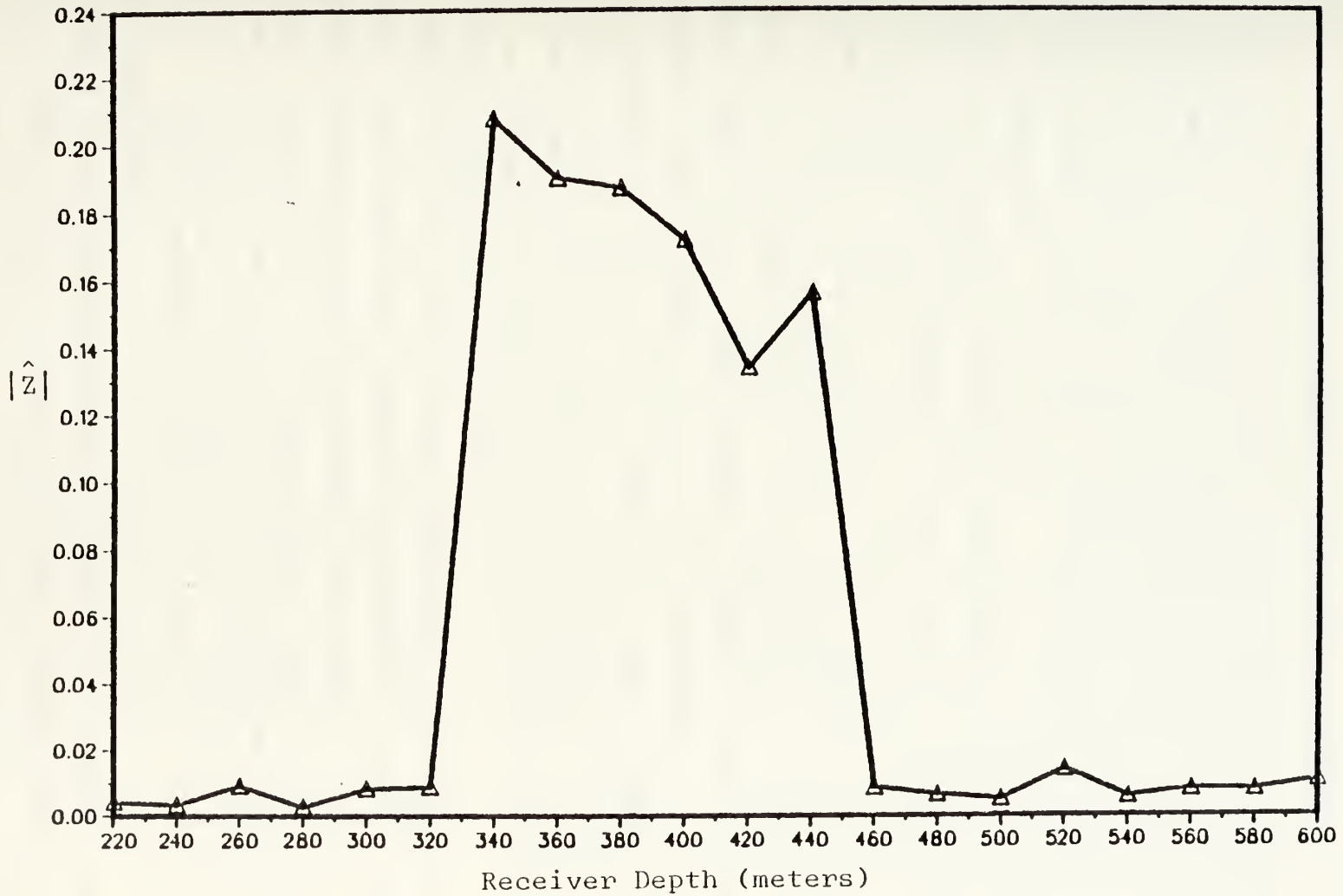

Figure 4.22 Beam Pattern (R=250 km, 20 depths, 5 receivers, source at 380 m)

V. DISCUSSION OF RESULTS AND RECOMMENDATIONS

For many of the cases examined, the linear minimum variance estimation technique gives high resolution in the "depth beam pattern" for sources at long range. Figures 4.2 and 4.3 represent beam patterns for four depths with two receivers where the main lobe is at the desired source depth with a beamwidth of 30 meters. For 20 depths with 2 receivers the system is highly overdetermined and the beam pattern portrayed in figure 4.4 has its main lobe 80 meters from the source depth. At the source depth the strength of the beam pattern is 3.6 db down. For figure 4.5 the main lobe is on the source depth with a beamwidth of 250 meters.

For all 20 source depths with 5 hydrophone receivers the results range from a beam pattern having its main lobe on the source depth (figure 4.7) with a beamwidth of 20 meters and a secondary lobe 12.1 db down to a beam pattern having its main lobe 40 meters from the source depth (figure 4.22). For the beam pattern in figure 4.22 the width of the main lobe is 80 meters and the strength of the beam pattern at the source depth is 0.8 db down.

When using the conventional beamformer, the beam pattern results with amplitude weights determined by the linear minimum variance estimation method were superior to the beam pattern determined by using unity amplitude weights. However in all cases the conventional beamformer was significantly inferior to the beam pattern determined by the linear minimum variance estimation technique in terms of depth resolution.

The linear minimum variance estimation technique could not be used for all ranges with the chosen sound speed profile. For ranges of 187 km and 235 km the relative

travel times produce an '$\underline{A}$' matrix which is so ill-conditioned that the 'IMSL' subroutine 'LINV2F' cannot determine an accurate inverse of '$\underline{A}^T\underline{A}$'.

There is sufficient evidence from this initial investigation that the linear minimum variance estimation technique applied to a long linear vertical array can yield high resolution depth information about passive sources at very long ranges. However, further investigation is needed before any field tests are in order. Recommendations for further research are:

1. The use of a more realistic speed of sound profile, preferably one actually characteristic of the 'SOFAR' channel.
2. The use of more hydrophones in the vertical array. As more receivers are used the beam pattern should approach the desired beam pattern more closely (a less overdetermined system should yield smaller total minimum mean square error).
3. Investigation into causes for the peak response falling at other than the desired depth and techniques for correcting this problem when using the linear minimum variance estimation technique.
4. Alternate assumption for choosing the ray paths to include in the '$\underline{A}$' matrix. For example one might choose the rays with the greatest intensity instead of the shortest travel time.
5. The possibility of estimating range as well as the depth of a passive source with linear minimum variance estimation techniques.

In conclusion, the linear minimum variance estimation technique of beamforming was significantly superior to the conventional beamformer. High resolution depth information about passive sources at long ranges is provided.

APPENDIX A

RELATIVE TRAVEL TIME CALCULATION

This program calculates the relative travel times from each source depth to the horizontal range and the depth of the ray at this range. To save paper, only the two shallowest source depths are listed.

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C     PROGRAM WHICH CALCULATES RELATIVE TRAVEL TIMES AND DEPTHS OF        THE00010
C     THESE RAYS AT THE SELECTED RANGE                                    THE00020
C                                                                         THE00030
C     DESCRIPTION OF IMPORTANT PARAMETERS                                 THE00040
C                                                                         THE00050
C*     F3 - MAXIMUM UPWARD ANGLE WHICH YIELDS R-R RAY                     THE00060
C*     CDEP - CHANGE IN VERTICAL DEPTH                                    THE00070
C*     RANGE - HORIZONTAL RANGE FROM SOURCE TO RECEIVER                   THE00080
C*     RAN - VALUE OF SUMMED HORIZONTAL RANGE OF PRESENT RAY              THE00090
C*     DEPTH - VALUE OF SUMMED DEPTH OF PRESENT RAY                       THE00100
C*     C1 - SPEED OF SOUND AT PRESENT DEPTH                               THE00110
C*     TIME - VALUE OF SUMMED TIME OF PRESENT RAY                         THE00120
C*     X1 - COSINE OF INITIAL DEPRESSION ANGLE OF ARC OF PRESENT RAY      THE00130
C*     Y1 - SINE OF INITIAL DEPRESSION ANGLE OF ARC OF PRESENT RAY        THE00140
C*     X2 - COSINE OF FINAL DEPRESSION ANGLE OF ARC OF PRESENT RAY        THE00150
C*     Y2 - SINE OF FINAL DEPRESSION ANGLE OF ARC OF PRESENT RAY          THE00160
C*     REFTI - TOTAL TRAVEL TIME OF THE REFERENCE RAY                     THE00170
C*     G - VELOCITY GRADIENT                                              THE00180
C                                                                         THE00190
C     THE REFERENCE RAY IS AT THE SHALLOWEST SOURCE DEPTH AND HAS AN      THE00200
C     INITIAL DEPRESSION ANGLE OF 0 DEGREES                               THE00210
C                                                                         THE00220
C     DEPRESSION ANGLES WHICH ARE DOWNWARD WITH RESPECT TO THE            THE00230
C     HORIZONTAL AXIS ARE POSITIVE                                        THE00240
C                                                                         THE00250
      DIMENSION X(600),Y(600)                                             THE00260
      DOUBLE PRECISION A1,X2,F2,C1,D,C,TIME,T,PI,Y1,X1,T1,G,Y2,F1,R6,P,   THE00270
     *R11,CDEP,RAN,RANGE,ANGLE,DEPTH,D1,DRAN,R2,R3,T3,ERAN,T4,FRAN,R4,   THE00280
     *R5,T5,T6,GRAN,HRAN,D2,D3,D4,D5,D6,R1A,DEGRE,X3,F3,D7,D8,D9,D10,    THE00290
     *T7,T8,T9,T10,RAN1,RAN2,RAN3,RAN4,TI,X,Y,RANIN,REFTI                THE00300
C                                                                         THE00310
C     SET INITIAL CONDITIONS                                              THE00320
C                                                                         THE00330
      RANIN=0.0D0                                                         THE00340
      K1=1                                                                THE00350
3     DEPTH=200.0D0                                                       THE00360
5     DEPTH=DEPTH+20.0D0                                                  THE00370
      RANGE=20000.0D0                                                     THE00380
      G=-0.01700                                                          THE00390
      I=0                                                                 THE00400
      N=0                                                                 THE00410
      K=0                                                                 THE00420
      RAN=0.000                                                           THE00430
      R1A=0.0D0                                                           THE00440
      ANGLE=0.0D0                                                         THE00450
      C1=1500.0D0+G*DEPTH                                                 THE00460
      A1=ANGLE                                                            THE00470
      R=RANGE                                                             THE00480
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      D=DEPTH                                                          THE0C490
      C=C1                                                             THE0C500
      TIME=0.CDO                                                       THE0C510
      T=TIME                                                           THE0C520
C                                                                      THE0C530
C    CALCULATE INITIAL MAXIMUM UPWARD DEPRESSICN ANGLE                 THE0C540
C                                                                      THE0C550
      X3=C1/(((G*(-D))+C1)                                             THE0C560
66    F3=-DAR(OS(X3)                                                   THE0C570
      WRITE(6,600)                                                     THE0C580
      WRITE(8,600)                                                     THE0C590
600   FORMAT(3X,'DEPTH',3X,'RELATIVE TRAVEL TIME',2X,'INITIAL ANGLE')  THE0C600
      PI=3.14159265358980                                              THE0C610
      P=PI/4.CDO                                                       THE0C620
C                                                                      THE0C630
C     CALCULATE HORIZONTAL RANGE CF ARC 1                              THE0C640
C                                                                      THE0C650
      Y1=DSIN(A1)                                                      THE0C660
      X1=DCOS(A1)                                                      THE0C670
88    G=-0.017DO                                                       THE0C680
      X2=(((1000.CDO-DEPTH)*G*X1)/C1)+X1                               THE0C690
      F2=DARCOS(X2)                                                    THE0C700
      Y2=DSIN(F2)                                                      THE0C710
      RAN=(C1*(Y1-Y2))/(G*X1)+RAN                                      THE0C720
      R1=RAN                                                           THE0C730
      IF(RANGE-RAN)11,11,13                                            THE0C740
C                                                                      THE0C750
C   IF HORIZONTAL RANGE OF ARC 1 IS LESS THAN TOTAL HORIZONTAL RANGE   THE0C760
C   THEN CONTINUE PROGRAM, OTHERWISE EXIT TO 11                        THE0C770
C                                                                      THE0C780
C     CALCULATE TRAVEL TIME OF ARC 1                                   THE0C790
C                                                                      THE0C800
13    IF(TIME)83,83,84                                                 THE0C810
83    TIME=(-1.0DC/G)*DLOG((DTAN(P+(F2/2.0DO)))/DTAN(P+(A1/2.0DO)))    THE0C820
      GO TO 79                                                         THE0C830
84    TIME=(-1.0DC/G)*DLOG((DTAN(P+(F2/2.0DO)))/DTAN(P))+TIME          THE0C840
C                                                                      THE0C850
C     CALCULATE AND ADD ARC 2 TO SUM OF PRESENT HORIZONTAL RANGE       THE0C860
C                                                                      THE0C870
79    T1=TIME                                                          THE0C880
      Y1=Y2                                                            THE0C890
      X1=X2                                                            THE0C900
      CDEP=1CC0.0CO-DEPTH                                              THE0C910
      C1=C1+G*CDEP                                                     THE0C920
      DEPTH=DEPTH+CDEP                                                 THE0C930
      D1=DEPTH                                                         THE0C940
      G=0.017DO                                                        THE0C950
      CDEP=C1*(1.CDO-X1)/(G*X1)                                        THE0C960
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      CRAN=(C1*Y1)/(G*X1)                                                THE00970
      RAN=RAN+CRAN                                                       THE00980
      R11=RAN                                                            THE00990
      IF(RANGE-RAN)12,12,14                                              THE01000
C                                                                        THE01010
CC    IF HORIZONTAL RANGE OF ARC 1 AND 2 IS LESS THAN THE TOTAL          THE01020
C     RANGE THEN CONTINUE PROGRAM, OTHERWISE EXIT TO 12                  THE01030
C                                                                        THE01040
C     CALCULATE SUMMED TRAVEL TIME                                       THE01050
C                                                                        THE01060
14    C1=C1+G*CDEP                                                       THE01070
      DEPTH=DEPTH+CDEP                                                   THE01080
      D2=DEPTH                                                           THE01090
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F2/2.0D0))))          THE01100
C                                                                        THE01110
CC    ADD ARC 3                                                          THE01120
C                                                                        THE01130
      T2=TIME                                                            THE01140
      CDEP=-CDEP                                                         THE01150
      X2=((CDEP*G)/C1)+1.0D0                                             THE01160
      F2=-DARCOS(X2)                                                     THE01170
      Y2=DSIN(F2)                                                        THE01180
      DRAN=(C1*(-Y2))/G                                                  THE01190
      RAN=RAN+DRAN                                                       THE01200
      R2=RAN                                                             THE01210
      IF(RANGE-RAN)21,21,22                                              THE01220
C                                                                        THE01230
CC    ADD ARC 4                                                          THE01240
C                                                                        THE01250
22    C1=C1+G*CDEP                                                       THE01260
      DEPTH=DEPTH+CDEP                                                   THE01270
      D3=DEPTH                                                           THE01280
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/(DTAN(P)))          THE01290
      T3=TIME                                                            THE01300
      G=-0.017D0                                                         THE01310
      X1=X2                                                              THE01320
      Y1=Y2                                                              THE01330
      CDEP=(C1*(1.0D0-X1))/(G*X1)                                        THE01340
      ERAN=C1*Y1/(G*X1)                                                  THE01350
      F1=DARSIN(Y1)                                                      THE01360
      RAN=RAN+ERAN                                                       THE01370
      R3=RAN                                                             THE01380
      IF(RANGE-RAN)41,41,42                                              THE01390
C                                                                        THE01400
CC    ADD ARC 5                                                          THE01410
C                                                                        THE01420
42    C1=C1+G*CDEP                                                       THE01430
      DEPTH=DEPTH+CDEP                                                   THE01440
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      D4=DEPTH                                                          THE01450
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F1/2.0D0))))         THE01460
      T4=TIME                                                           THE01470
      CDEP=-CDEP                                                        THE01480
      X2=((CDEP*G)/C1)+1.0D0                                            THE01490
      F2=DARCOS(X2)                                                     THE01500
      Y2=DSIN(F2)                                                       THE01510
      FRAN=(C1*(-Y2))/G                                                 THE01520
      RAN=RAN+FRAN                                                      THE01530
      R4=RAN                                                            THE01540
      IF(RANGE-RAN)51,51,52                                             THE01550
C                                                                       THE01560
C     ADD ARC 6                                                         THE01570
C                                                                       THE01580
52    C1=C1+G*CDEP                                                      THE01590
      DEPTH=DEPTH+CDEP                                                  THE01600
      D5=DEPTH                                                          THE01610
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P+F2/2.0D0))/(DTAN(P)))           THE01620
      T5=TIME                                                           THE01630
      X1=X2                                                             THE01640
      Y1=Y2                                                             THE01650
      G=0.017DC                                                         THE01660
      CDEP=(C1*(1.0D0-X1))/(G*X1)                                       THE01670
      GRAN=(C1*Y1)/(G*X1)                                               THE01680
      RAN=GRAN+RAN                                                      THE01690
      R5=RAN                                                            THE01700
      IF(RANGE-RAN)61,61,62                                             THE01710
C                                                                       THE01720
C     ADD ARC 7                                                         THE01730
C                                                                       THE01740
62    C1=C1+G*CDEP                                                      THE01750
      DEPTH=DEPTH+CDEP                                                  THE01760
      D6=DEPTH                                                          THE01770
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F2/2.0D0))))         THE01780
      T6=TIME                                                           THE01790
      CDEP=-CDEP                                                        THE01800
      X2=(CDEP*G)/C1+1.0D0                                              THE01810
      F2=-DARCOS(X2)                                                    THE01820
      Y2=DSIN(F2)                                                       THE01830
      HRAN=(C1*(-Y2))/G                                                 THE01840
      RAN=RAN+HRAN                                                      THE01850
      R6=RAN                                                            THE01860
      IF(RANGE-RAN)71,71,92                                             THE01870
C                                                                       THE01880
C     ADD ARC 8                                                         THE01890
C                                                                       THE01900
92    C1=C1+G*CDEP                                                      THE01910
      DEPTH=DEPTH+CDEP                                                  THE01920
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      D7=DEPTH                                                          THE01930
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/(DTAN(P)))         THE01940
      T7=TIME                                                           THE01950
      G=-0.017D0                                                        THE01960
      X1=X2                                                             THE01970
      Y1=Y2                                                             THE01980
      CDEP=(C1*(1.0D0-X1))/(G*X1)                                       THE01990
      RAN1=C1*Y1/(G*X1)                                                 THE02000
      F1=DARSIN(Y1)                                                     THE02010
      RAN=RAN+RAN1                                                      THE02020
      R7=RAN                                                            THE02030
      IF(RANGE-RAN)101,101,102                                          THE02040
C                                                                       THE02050
C     ADD ARC 9                                                         THE02060
C                                                                       THE02070
102   C1=C1+G*CDEP                                                      THE02080
      DEPTH=DEPTH+CDEP                                                  THE02090
      D8=DEPTH                                                          THE02100
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F1/2.0D0))))         THE02110
      T8=TIME                                                           THE02120
      CDEP=-CDEP                                                        THE02130
      X2=((CDEP*G)/C1)+1.0D0                                            THE02140
      F2=DARCOS(X2)                                                     THE02150
      Y2=DSIN(F2)                                                       THE02160
      RAN2=(C1*(-Y2))/G                                                 THE02170
      RAN=RAN+RAN2                                                      THE02180
      R8=RAN                                                            THE02190
      IF(RANGE-RAN)111,111,112                                          THE02200
C                                                                       THE02210
C     ADD ARC 10                                                        THE02220
C                                                                       THE02230
112   C1=C1+G*CDEP                                                      THE02240
      DEPTH=DEPTH+CDEP                                                  THE02250
      D9=DEPTH                                                          THE02260
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P+F2/2.0D0))/(DTAN(P)))           THE02270
      T9=TIME                                                           THE02280
      X1=X2                                                             THE02290
      Y1=Y2                                                             THE02300
      G=0.017D0                                                         THE02310
      CDEP=(C1*(1.0D0-X1))/(G*X1)                                       THE02320
      RAN3=(C1*Y1)/(G*X1)                                               THE02330
      RAN=RAN+RAN3                                                      THE02340
      R9=RAN                                                            THE02350
      IF(RANGE-RAN)121,121,122                                          THE02360
C                                                                       THE02370
C     ADD ARC 11                                                        THE02380
C                                                                       THE02390
122   C1=C1+G*CDEP                                                      THE02400
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      DEPTH=DEPTH+CDEP                                                  THE02410
      D10=DEPTH                                                         THE02420
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F2/2.0D0))))         THE02430
      T10=TIME                                                          THE02440
      CDEP=-CDEP                                                        THE02450
      X2=(CDEP*G)/C1+1.0D0                                              THE02460
      F2=-DARCOS()2)                                                    THE02470
      Y2=DSIN(F2)                                                       THE02480
      RAN4=(C1*(-Y2))/G                                                 THE02490
      RAN=RAN+RAN4                                                      THE02500
      R6=RAN                                                            THE02510
      IF(RANGE-RAN)131,131,92                                           THE02520
C                                                                       THE02530
C     CONTINUE ADDING ARCS IF SUMMED HORIZONTAL RANGE IS LESS THAN      THE02540
C     TOTAL RANGE, OTHERWISE EXIT TO 131                                THE02550
C                                                                       THE02560
C                                                                       THE02570
C     CALCULATES TRAVEL TIME OF REFERENCE RAY                           THE02580
C                                                                       THE02590
11    Y2=Y1-((RANGE-R1A)*G*X1)/C1                                       THE02600
      F2=DARSIN(Y2)                                                     THE02610
      X2=DCOS(F2)                                                       THE02620
      CDEP=C1*(X2-X1)/(G*X1)                                            THE02630
      DEPTH=DEPTH+CDEP                                                  THE02640
      IF(TIME)7,7,8                                                     THE02650
8     TIME=(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))+TIME           THE02660
      GO TO 32                                                          THE02670
7     TIME=(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/(DTAN(P+(A1/2.0D0))))   THE02680
32    DEGRE=A1*180.0D0/PI                                               THE02690
      N=N+1                                                             THE02700
      IF(A1)72,29,72                                                    THE02710
29    K1=K1-1                                                           THE02720
      IF(K1)72,811,72                                                   THE02730
811   REFTI=TIME                                                        THE02740
      GO TO 72                                                          THE02750
C                                                                       THE02760
C     SUBROUTINE TO CALCULATE DEPTH AND TIME IF HORIZONTAL RANGE OF     THE02770
C     ARC 1 IS GREATER THAN SOURCE-RECEIVER RANGE                       THE02780
C                                                                       THE02790
12    F1=F2                                                             THE02800
      Y2=Y1-((RANGE-R1)*G*X1)/C1                                        THE02810
      F2=DARSIN(Y2)                                                     THE02820
      X2=DCOS(F2)                                                       THE02830
      CDEP=C1*(X2-X1)/(G*X1)                                            THE02840
      TIME=T1+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))  THE02850
      DEPTH=D1+CDEP                                                     THE02860
      GO TO 32                                                          THE02870
C                                                                       THE02880
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C       SUBROUTINE TO CALCULATE DEPTH AND TIME IF ADDITION OF RANGE OF     THE02890
C       ARC 2 CAUSES SUMMED RANGE TO BE GREATER THAN SOURCE-RECEIVER       THE02900
C       RANGE                                                              THE02910
C                                                                          THE02920
21      Y2=-(RANGE-R11)*G/C1                                               THE02930
        F2=DARSIN(Y2)                                                      THE02940
        X2=DCOS(F2)                                                        THE02950
        CDEP=C1*(X2-1.0D0)/G                                               THE02960
        TIME=T2+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))              THE02970
        DEPTH=D2+CDEP                                                      THE02980
        GO TO 32                                                           THE02990
C                                                                          THE03000
C       USE IF ARC 3 CAUSES OVERFLOW                                       THE03010
C                                                                          THE03020
41      Y2=Y1-((RANGE-R2)*G*X1)/C1                                         THE03030
        F2=DARSIN(Y2)                                                      THE03040
        X2=DCOS(F2)                                                        THE03050
        CDEP=C1*(X2-X1)/(G*X1)                                             THE03060
        TIME=T3+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))   THE03070
        DEPTH=D3+CDEP                                                      THE03080
        GO TO 32                                                           THE03090
C                                                                          THE03100
C       USE IF ARC 4 CAUSES OVERFLOW                                       THE03110
C                                                                          THE03120
51      Y2=-(RANGE-R3)*G/C1                                                THE03130
        F2=DARSIN(Y2)                                                      THE03140
        X2=DCOS(F2)                                                        THE03150
        CDEP=C1*(X2-1.0D0)/G                                               THE03160
        TIME=T4+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))              THE03170
        DEPTH=D4+CDEP                                                      THE03180
        GO TO 32                                                           THE03190
C                                                                          THE03200
C       USE IF ARC 5 CAUSES OVERFLOW                                       THE03210
C                                                                          THE03220
61      F1=F2                                                              THE03230
        Y2=Y1-((RANGE-R4)*G*X1)/C1                                         THE03240
        F2=DARSIN(Y2)                                                      THE03250
        X2=DCOS(F2)                                                        THE03260
        CDEP=C1*(X2-X1)/(G*X1)                                             THE03270
        TIME=T5+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))   THE03280
        DEPTH=D5+CDEP                                                      THE03290
        GO TO 32                                                           THE03300
C                                                                          THE03310
C       USE IF ARC 6 CAUSES OVERFLOW                                       THE03320
C                                                                          THE03330
71      Y2=-(RANGE-R5)*G/C1                                                THE03340
        F2=DARSIN(Y2)                                                      THE03350
        X2=DCOS(F2)                                                        THE03360
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      CDEP=C1*(X2-1.0D0)/G                                                    THE03370
      TIME=T6+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))                   THE03380
      DEPTH=D6+CDEP                                                           THE03390
      GO TO 32                                                                THE03400
C                                                                             THE03410
C     USE IF ARC 7 CAUSES OVERFLOW                                            THE03420
C                                                                             THE03430
101   Y2=Y1-((RANGE-R6)*G*X1)/C1                                              THE03440
      F2=DARSIN(Y2)                                                           THE03450
      X2=DCOS(F2)                                                             THE03460
      CDEP=C1*(X2-X1)/(G*X1)                                                  THE03470
      TIME=T7+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))        THE03480
      DEPTH=D7+CDEP                                                           THE03490
      GO TO 32                                                                THE03500
C                                                                             THE03510
C     USE IF ARC 8 CAUSES OVERFLOW                                            THE03520
C                                                                             THE03530
111   Y2=-(RANGE-R7)*G/C1                                                     THE03540
      F2=DARSIN(Y2)                                                           THE03550
      X2=DCOS(F2)                                                             THE03560
      CDEP=C1*(X2-1.0D0)/G                                                    THE03570
      TIME=T8+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))                   THE03580
      DEPTH=D8+CDEP                                                           THE03590
      GO TO 32                                                                THE03600
C                                                                             THE03610
C     USE IF ARC 9 CAUSES OVERFLOW                                            THE03620
C                                                                             THE03630
121   F1=F2                                                                   THE03640
      Y2=Y1-((RANGE-R8)*G*X1)/C1                                              THE03650
      F2=DARSIN(Y2)                                                           THE03660
      X2=DCOS(F2)                                                             THE03670
      CDEP=C1*(X2-X1)/(G*X1)                                                  THE03680
      TIME=T9+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))        THE03690
      DEPTH=D9+CDEP                                                           THE03700
      GO TO 32                                                                THE03710
C                                                                             THE03720
C     USE IF ARC 10 CAUSES OVERFLOW                                           THE03730
C                                                                             THE03740
131   Y2=-(RANGE-R9)*G/C1                                                     THE03750
      F2=DARSIN(Y2)                                                           THE03760
      X2=DCOS(F2)                                                             THE03770
      CDEP=C1*(X2-1.0D0)/G                                                    THE03780
      TIME=T10+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))                  THE03790
      DEPTH=D10+CDEP                                                          THE03800
      GO TO 32                                                                THE03810
C                                                                             THE03820
C     DEPTH, RELATIVE TIME, AND INITIAL DEPRESSION ANGLE OF RAY ARE           THE03830
C     PUT IN A LISTING                                                        THE03840
```

```
C                                                                       THE03850
72    K=K+1                                                             THE03860
      Y(K)=TIME-REFTI                                                   THE03870
      X(K)=DEPTH                                                        THE03880
      WRITE(6,901)X(K),Y(K),DEGRE                                       THE03890
      WRITE(8,901)X(K),Y(K),DEGRE                                       THE03900
901   FORMAT(F10.4,F14.7,9X,F12.8)                                      THE03910
C                                                                       THE03920
C     INCREMENT THE INITIAL DEPRESSION ANGLE BY 0.1 DEGREES AND RESET   THE03930
C     INITIAL CONDITIONS.  REPEAT THE OPERATION UNTIL MAXIMUM R-R RAY    THE03940
C     INITIAL DEPRESSION ANGLE IS REACHED. LAST RAY FROM SOURCE DEPTH   THE03950
C     IS AT THIS ANGLE.                                                 THE03960
C                                                                       THE03970
      IF(A1)98,73,73                                                    THE03980
73    A1=A1+(PI*0.1D0)/180.0D0                                          THE03990
      RANGE=R                                                           THE04000
      DEPTH=D                                                           THE04010
      C1=C                                                              THE04020
      RAN=0.0D0                                                         THE04030
      TIME=0.0D0                                                        THE04040
      Y1=DSIN(A1)                                                       THE04050
      X1=DCOS(A1)                                                       THE04060
      IF(A1+F3)88,88,97                                                 THE04070
97    I=I+1                                                             THE04080
      A1=-F3                                                            THE04090
      Y1=DSIN(A1)                                                       THE04100
      X1=DCOS(A1)                                                       THE04110
      IF(I-2)98,99,99                                                   THE04120
C                                                                       THE04130
C                                                                       THE04140
C     THE FOLLOWING SUBROUTINE IS FOR NEGATIVE (UPWARD) INITIAL         THE04150
C     DEPRESSION ANGLES                                                 THE04160
C                                                                       THE04170
96    A1=ANGLE                                                          THE04180
98    RANGE=R                                                           THE04190
      DEPTH=D                                                           THE04200
      C1=C                                                              THE04210
      A1=A1-(PI*0.1D0)/180.0D0                                          THE04220
      Y1=DSIN(A1)                                                       THE04230
      X1=DCOS(A1)                                                       THE04240
      IF(A1+(-F3))93,53,53                                              THE04250
 93   I=I+1                                                             THE04260
      A1=F3                                                             THE04270
      Y1=DSIN(A1)                                                       THE04280
      X1=DCOS(A1)                                                       THE04290
      IF(I-4)53,99,99                                                   THE04300
53    G=-0.017D0                                                        THE04310
      R1A=C1*Y1/(G*X1)                                                  THE04320
```

```
      IF(RANGE-R1A)81,81,82                                                    THE04330
82    CDEP=C1*(1.0D0-X1)/(G*X1)                                                THE04340
      RAN=R1A                                                                  THE04350
      TIME=(-1.0D0/G)*DLOG((DTAN(P))/DTAN(P+(A1/2.0D0)))                       THE04360
      X1=1.0D0                                                                 THE04370
      Y1=0.0D0                                                                 THE04380
      DEPTH=CDEP+DEPTH                                                         THE04390
      C1=C1+G*CDEP                                                             THE04400
      GO TO 88                                                                 THE04410
81    Y2=Y1-(RANGE*G*X1)/C1                                                    THE04420
      F2=DARSIN(Y2)                                                            THE04430
      X2=DCOS(F2)                                                              THE04440
      TIME=(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(A1/2.0D0)))            THE04450
      CDEP=C1*(X2-X1)/(G*X1)                                                   THE04460
      DEPTH=DEPTH+CDEP                                                         THE04470
      GO TO 32                                                                 THE04480
C                                                                              THE04490
C     K IS THE NUMBER OF RAYS CALCULATED FOR EACH SOURCE DEPTH                 THE04500
C                                                                              THE04510
99    CONTINUE                                                                 THE04520
      WRITE(6,991)DEPTH,K                                                      THE04530
      WRITE(8,991)DEPTH,K                                                      THE04540
991   FORMAT(1X,'NUMBER OF RAYS FOR THE ',F7.2,' METER DEPTH IS',1X,I3)        THE04550
      WRITE(6,178)                                                             THE04560
      WRITE(8,178)                                                             THE04570
178   FORMAT(1X,'                ',1X)                                         THE04580
531   CONTINUE                                                                 THE04590
C                                                                              THE04600
C     INCREMENT THE SOURCE DEPTH 20 METERS UNTIL 600 METERS                    THE04610
C     IS EXCEEDED                                                              THE04620
C                                                                              THE04630
      IF(DEPTH-600.0D0)5,9,9                                                   THE04640
9     CONTINUE                                                                 THE04650
      STOP                                                                     THE04660
      END                                                                      THE04670
                                                                               THE04680
   DEPTH    RELATIVE TRAVEL TIME   INITIAL ANGLE                               THE04690
 1157.4079        0.0                  0.0                                     THE04700
 1173.6307        0.0012725            0.10000000                              THE04710
 1185.6802        0.0022367            0.20000000                              THE04720
 1193.6744        0.0028523            0.30000000                              THE04730
 1197.7002        0.0031453            0.40000000                              THE04740
 1197.8063        0.0031655            0.50000000                              THE04750
 1193.9812        0.0028606            0.60000000                              THE04760
 1186.1832        0.0022450            0.70000000                              THE04770
 1174.3260        0.0013153            0.80000000                              THE04780
 1158.2790        0.0000505            0.90000000                              THE04790
 1137.8606       -0.0016306            1.00000000                              THE04800
```

1112.8688	-0.C037000	1.10000000	THE04810
1083.0441	-0.C062330	1.20000000	THE04820
1048.1012	-0.C092386	1.30000000	THE04830
1007.6893	-0.C128722	1.40000000	THE04840
962.3588	-0.C169717	1.50000000	THE04850
913.8646	-0.C212090	1.60000000	THE04860
862.6216	-0.C255669	1.70000000	THE04870
809.0476	-0.C299738	1.80000000	THE04880
753.5937	-0.C343231	1.90000000	THE04890
696.7140	-0.C386434	2.00000000	THE04900
638.9217	-0.C427604	2.10000000	THE04910
580.7177	-0.C467210	2.20000000	THE04920
522.6453	-0.C504554	2.30000000	THE04930
465.2722	-0.C538628	2.40000000	THE04940
409.1732	-0.C569386	2.50000000	THE04950
354.9462	-0.C596485	2.60000000	THE04960
303.2019	-0.C620132	2.70000000	THE04970
254.5751	-0.C639836	2.80000000	THE04980
209.7138	-0.C655253	2.90000000	THE04990
169.2698	-0.C667447	3.00000000	THE05000
133.9208	-0.C675970	3.10000000	THE05010
104.3509	-0.C680958	3.20000000	THE05020
81.2518	-0.C683973	3.30000000	THE05030
65.3323	-0.C684673	3.40000000	THE05040
57.3071	-0.C684959	3.50000000	THE05050
57.9020	-0.C685065	3.60000000	THE05060
67.8515	-0.C686560	3.70000000	THE05070
87.8990	-0.C690506	3.80000000	THE05080
118.7971	-0.C699827	3.90000000	THE05090
161.3067	-0.C715197	4.00000000	THE05100
185.4389	-0.C725201	4.04685864	THE05110

NUMBER OF RAYS FOR THE 220.00 METER DEPTH IS 42 THE05120

THE05130

DEPTH	RELATIVE TRAVEL TIME	INITIAL ANGLE	
1420.4553	0.C184080	0.0	THE05140
1432.3934	0.C190909	0.10000000	THE05150
1441.2554	0.C195765	0.20000000	THE05160
1447.1769	0.C199448	0.30000000	THE05170
1450.2256	0.C200907	0.40000000	THE05180
1450.4492	0.C201025	0.50000000	THE05190
1447.8389	0.C199679	0.60000000	THE05200
1442.3401	0.C196584	0.70000000	THE05210
1433.8582	0.C191709	0.80000000	THE05220
1422.2537	0.C184861	0.90000000	THE05230
1407.3428	0.C175682	1.00000000	THE05240
1388.9098	0.C164489	1.10000000	THE05250
1366.6786	0.C150164	1.20000000	THE05260
1340.3486	0.C132680	1.30000000	THE05270

1309.5720	0.C111410	1.400C0000	THE05290
1273.9739	0.C086388	1.50000000	THE05300
1233.1130	0.C055768	1.60CC0000	THE05310
1186.5432	0.C019620	1.70000000	THE05320
1133.7675	-0.C023034	1.80000000	THE05330
1074.2521	-0.C073385	1.90000000	THE05340
1007.4265	-0.C132848	2.00000000	THE05350
935.3854	-0.C197747	2.10000000	THE05360
862.1978	-0.C260831	2.20000000	THE05370
788.4706	-0.C320888	2.30000000	THE05380
714.7754	-0.C378035	2.40000000	THE05390
641.7250	-0.C431235	2.50000000	THE05400
569.9417	-0.C479999	2.60000000	THE05410
500.0637	-0.C523972	2.70000000	THE05420
432.7447	-0.C562938	2.80000000	THE05430
368.6572	-0.C596407	2.90000000	THE05440
308.4811	-0.C624466	3.00000000	THE05450
252.9088	-0.C647361	3.10000000	THE05460
202.6509	-0.C664672	3.20000000	THE05470
158.4175	-0.C677407	3.30000000	THE05480
120.9356	-0.C685919	3.40000000	THE05490
90.9398	-0.C690749	3.50000000	THE05500
69.1725	-0.C692636	3.60000000	THE05510
56.3833	-0.C693339	3.70000000	THE05520
53.3308	-0.C693250	3.80000000	THE05530
60.7798	-0.C694228	3.90000000	THE05540
79.5029	-0.C697576	4.00000000	THE05550
110.2802	-0.C706456	4.10000000	THE05560
153.8989	-0.C721825	4.20000000	THE05570
167.9190	-0.C727444	4.22688592	THE05580
NUMBER OF RAYS FOR THE 240.00 METER DEPTH IS 44			THE05590
			THE05600

APPENDIX B

INTERPOLATION OF RELATIVE TRAVEL TIME CALCULATIONS

This program does an interpolation of the output data generated in Appendix A. The relative travel times are interpolated for the receiving hydrophone depths.

```
C     PROGRAM WHICH INTERPOLATES RELATIVE TRAVEL TIMES TO                    TRU0C010
C     RECEIVER DEPTHS                                                       TRU0C020
C                                                                           TRU0C030
C      DESCRIPTION OF IMPORTANT PARAMETERS                                  TRU0C040
C                                                                           TRU00050
C*      K - NUMBER OF RAYS FOR EACH DEPTH                                   TRU0C060
C*      X - DEPTH WHERE RAY REACHES HORIZONTAL RANGE                        TRU00070
C*      Y - RELATIVE TRAVEL TIME OF EACH RAY                                TRU0C080
C*      U - RECEIVING HYDROPHONE DEPTH                                      TRU0C090
C*      S - RELATIVE TRAVEL TIME TO EACH HYDROPHONE                         TRU00100
C                                                                           TRU0C110
C                                                                           TRU0C120
$JOB            14,3P,TIME=56                                               TRU00130
      DIMENSION X(68),Y(68),IR(68),WK(68),C(67,3),IC(67),U(5),S(5),K(20)    TRU0C140
      IJ=0                                                                  TRU00150
2     IJ=IJ+1                                                               TRU0C160
C                                                                           TRU0C170
C      R,X,AND Y ARE THE OUTPUTS FROM THE PROGRAM IN APPENDIX A.            TRU0C180
C     THESE ARE THE INPUTS TO THIS PROGRAM.  NOTE THAT WHEN MORE THAN       TRU0C190
C     ONE RAY TERMINATES AT THE SAME DEPTH FOR A GIVEN SOURCE DEPTH         TRU00200
C     THEN THE RAY WHICH ARRIVES FIRST IS USED IN THE INTERPOLATION         TRU0C210
C     PROGRAM.  TO INSURE THAT THE OTHER NONAPPLICABLE RAYS DO NOT          TRU0C220
C     INTERFERE IN THE INTERPOLATION PROCESS THEIR TERMINATING DEPTHS       TRU0C230
C     ARE MADE ARBITRARILY LARGE                                            TRU0C240
C                                                                           TRU0C250
      READ(5,88)K(IJ)                                                       TRU0C260
88    FORMAT(I3)                                                            TRU0C270
      NX=K(IJ)                                                              TRU0C280
      DO 100 I=1,NX                                                         TRU00290
      READ(5,10)X(I),Y(I)                                                   TRU0C300
10    FORMAT(F10.4,F14.7)                                                   TRU0C310
100   CONTINUE                                                              TRU0C320
      K1=K(IJ)-1                                                            TRU0C330
      I1=0                                                                  TRU0C340
6     I1=I1+1                                                               TRU0C350
      IR(I1)=I1                                                             TRU0C360
      IF(I1-NX)6,9,9                                                        TRU00370
9     CONTINUE                                                              TRU0C380
C                                                                           TRU0C390
C      INPUT VECTORS TO BE INTERPOLATED MUST BE IN ASCENDING ORDER.         TRU00400
C      IMSL ROUTINES 'VSRTR' AND 'VSRTU' DO THIS.  IMSL ROUTINE             TRU0C410
C      'ICSCCU' INTERPOLATES THE SLICE COEFFICIENT                          TRU0C420
C                                                                           TRU0C430
      CALL VSRTR(X,NX,IR)                                                   TRU0C440
      CALL VSRTU(Y,1,1,NX,0,IR,WK)                                          TRU00450
      CALL ICSCCU(X,Y,NX,C,K1,IER)                                          TRU0C460
C                                                                           TRU0C470
C     RECEIVING HYDROPHONE DEPTHS ARE SELECTED.  IMSL ROUTINE 'ICSEVU'      TRU0C480
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C       EVALUATES THE SLICE AT THESE POINTS                                    TRUOC490
C                                                                              TRUOC500
        U(1)=100.0                                                             TRUOC510
        U(2)=200.0                                                             TRUOC520
        U(3)=300.0                                                             TRUOC530
        U(4)=400.0                                                             TRUOC540
        U(5)=500.0                                                             TRUOC550
        CALL ICSEVU(X,Y,NX,C,K1,U,S,5,IER)                                     TRUOC560
C                                                                              TRUOC570
C       OUTPUT ARE TRAVEL TIMES PRINTED BY ROW.  THE FIRST ROW IS FOR          TRUOC580
C       THE SHALLOWEST SOURCE DEPTH AND LAST ROW IS FOR THE DEEPEST            TRUOC590
C       SOURCE DEPTH.  THE FIRST COLUMN IS FOR THE SHALLOWEST RECEIVING        TRUOC600
C       HYDROPHONE AND THE FINAL COLUMN IS FOR THE DEEPEST HYDROPHONE.         TRUOC610
C                                                                              TRUOC620
        WRITE(6,101)(S(I),I=1,5)                                               TRUOC630
        WRITE(7,101)(S(I),I=1,5)                                               TRUOC640
101     FORMAT(5F14.7)                                                         TRUOC650
C                                                                              TRUOC660
C       PROGRAM TERMINATES WHEN NUMBER OF RAYS FOR A SOURCE DEPTH EQUALS       TRUOC670
C       THE NUMBER OF RAYS FOR THE DEEPEST SOURCE DEPTH                        TRUOC680
C                                                                              TRUOC690
        IF(K(IJ)-68)2,21,21                                                    TRUOC700
21      CONTINUE                                                               TRUOC710
        STOP                                                                   TRUOC720
        END                                                                    TRUOC730
$ENTRY                                                                         TRUOC740
                                                                               TRUOC750
    -0.0654169    -0.0658337    -0.0621516    -0.0574149    -0.0518334         TRUOC760
    -0.0703276    -0.0665503    -0.0628172    -0.0580503    -0.0524011         TRUOC770
    -0.0713123    -0.0673456    -0.0635689    -0.0587241    -0.0530590         TRUOC780
    -0.0723230    -0.0681941    -0.0643641    -0.0595105    -0.0537945         TRUOC790
    -0.0733907    -0.0690646    -0.0652127    -0.0603237    -0.0546137         TRUOC800
    -0.0745426    -0.0699940    -0.0661212    -0.0612039    -0.0554598         TRUOC810
    -0.0756658    -0.0709912    -0.0670775    -0.0621397    -0.0563615         TRUOC820
    -0.0747207    -0.0720147    -0.0680565    -0.0631501    -0.0572667         TRUOC830
    -0.0758055    -0.0731056    -0.0691335    -0.0641502    -0.0583075         TRUOC840
    -0.0769579    -0.0742301    -0.0701951    -0.0652131    -0.0593780         TRUOC850
    -0.0781758    -0.0753760    -0.0713190    -0.0663189    -0.0604718         TRUOC860
    -0.0793883    -0.0765576    -0.0724953    -0.0674813    -0.0616073         TRUOC870
    -0.0806178    -0.0777900    -0.0737091    -0.0686366    -0.0627376         TRUOC880
    -0.0819192    -0.0790446    -0.0749412    -0.0698603    -0.0639641         TRUOC890
    -0.0832829    -0.0803677    -0.0762265    -0.0711268    -0.0652111         TRUOC900
    -0.0846354    -0.0817073    -0.0775561    -0.0724360    -0.0664785         TRUOC910
    -0.0860246    -0.0830759    -0.0788832    -0.0737404    -0.0677993         TRUOC920
    -0.0874630    -0.0844590    -0.0802819    -0.0751390    -0.0691366         TRUOC930
    -0.0888928    -0.0859060    -0.0816601    -0.0764943    -0.0705184         TRUOC940
    -0.0902550    -0.0873991    -0.0830757    -0.0779512    -0.0719424         TRUOC950
```

APPENDIX C

RESULTING BEAM PATTERN FOR CALCULATED WEIGHTS

This program uses the output of Appendix B to calculate the '$\underline{A}$' matrix. From this, the amplitude and phase weights are determined by the linear minumum variance estimation technique. These weights are applied to the array and the beam pattern is obtained.

```
C      PROGRAM WHICH DOES THE FOLLOWING TASKS:                                  MA900010
C      * CALCULATES THE 'A' MATRIX FROM RELATIVE TRAVEL TIMES                   MA900020
C      * CALCULATES ANS WHICH REPRESENTS THE UNKNOWN - THE AMPLITUDE            MA900030
C        AND PHASE WEIGHTS TO BE APPLIED TO THE ARRAY                          MA900040
C      * USES 'ANS' AND 'A' TO CALCULATE THE RESULTING BEAM PATTERN             MA900050
C                                                                               MA900060
C                                                                               MA900070
C      DESCRIPTION OF IMPORTANT PARAMETERS                                      MA900080
C                                                                               MA900090
C      FREQ - FREQUENCY IN HZ.                                                  MA900100
C      Z - DESIRED BEAM PATTERN (REAL AND IMAGINARY PARTS SEPARATE)             MA900110
C      ANS - THE UNKNOWN, PHASE AND AMPLITUDE WEIGHTS DETERMINED BY             MA900120
C            LINEAR MINIMUM VARIANCE METHOD                                     MA900130
C      Y - RESULTING BEAM PATTERN (REAL AND IMAGINARY PARTS SEPARATE)           MA900140
C      XX - RESULTING BEAM PATTERN (REAL AND IMAGINARY PARTS COMBINED)          MA900150
C      F - RELATIVE TRAVEL TIMES CALCULATED IN APPENDIX B                       MA900160
C      W - WEIGHTING MATRIX                                                     MA900170
C      X1 - SOURCE DEPTH                                                        MA900180
C                                                                               MA900190
C                                                                               MA900200
$JOB              1493P,TIME=56                                                 MA900210
      DIMENSION A(40,10),F(20,5),ANG(20,5),W(40,40),Z(40,1),C(10,40),           MA900220
     *D(10,10),E(10,1),DINV(10,10),WKAREA(150),ANS(10,1),                       MA900230
     *B(20,20),Y(40,1),XX(20,1),X1(20,1)                                        MA900240
C                                                                               MA900250
CC     SET INITIAL CONDITIONS                                                   MA900260
C                                                                               MA900270
      J1=0                                                                      MA900280
      I1=0                                                                      MA900290
      I20=0                                                                     MA900300
      I19=0                                                                     MA900310
      I6=0                                                                      MA900320
      I5=0                                                                      MA900330
      J5=0                                                                      MA900340
      PI=3.14159265358980                                                       MA900350
      FREQ=100.00                                                               MA900360
      P=2.0*PI*FREQ                                                             MA900370
C                                                                               MA900380
CCC    READ IN THE RELATIVE TRAVEL TIMES DERIVED IN APPENDIX B                  MA900390
C                                                                               MA900400
      READ(5,10)((F(I,J),J=1,5),I=1,20)                                         MA900410
10    FORMAT(5F14.7)                                                            MA900420
C                                                                               MA900430
C      CALCULATE THE 'A' MATRIX.                                                MA900440
C      IF THERE IS NOT A SOUND SIGNAL AT A RECEIVER DEPTH THEN SET THE          MA900450
C      SINE AND COSINE TO ZERO.                                                 MA900460
C                                                                               MA900470
8     J1=J1+1                                                                   MA900480
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```
7       I1=I1+1                                                          MA90C490
        IF(F(I1,J1).EQ.0)GO TO 6                                         MA90C500
        ANG(I1,J1)=P*F(I1,J1)                                            MA90C510
        A(I1,J1)=COS(ANG(I1,J1))                                         MA90C520
        A(I1,J1+5)=SIN(ANG(I1,J1))                                       MA90C530
        A(I1+20,J1)=-A(I1,J1+5)                                          MA90C540
        A(I1+20,J1+5)=A(I1,J1)                                           MA90C550
        IF(I1-20)7,2,2                                                   MA90C560
2       I1=0                                                             MA90C570
        IF(J1-5)8,72,72                                                  MA90C580
6       A(I1,J1)=0                                                       MA90C590
        A(I1,J1+5)=0                                                     MA90C600
        A(I1+20,J1)=0                                                    MA90C610
        A(I1+20,J1+5)=0                                                  MA90C620
        IF(I1-20)7,2,2                                                   MA90C630
C                                                                        MA90C640
C       SET WEIGHTING MATRIX TO IDENTITY MATRIX.                         MA90C650
C       SET DESIRED BEAM PATTERN 'Z'.                                    MA90C660
C                                                                        MA90C670
72      I5=I5+1                                                          MA90C680
52      J5=J5+1                                                          MA90C690
        W(I5,J5)=0.0                                                     MA90C700
        IF(J5-40)52,53,53                                                MA90C710
53      J5=0                                                             MA90C720
        IF(I5-40)72,75,75                                                MA90C730
75      I6=I6+1                                                          MA90C740
        Z(I6,1)=0.0                                                      MA90C750
        W(I6,I6)=1.00                                                    MA90C760
        IF(I6-40)75,76,76                                                MA90C770
76      W(1,1)=1.00                                                      MA90C780
        Z(1,1)=1.00                                                      MA90C790
C                                                                        MA90C800
C       THE ENERGY SOURCE IS SELECTED TO BE ON THE SHALLOWEST DEPTH      MA90C810
C                                                                        MA90C820
C       THE IMSL ROUTINE SUMMARY                                         MA90C830
C       * VMULFM - TRANSPOSE OF FIRST MATRIX TIMES SECOND MATRIX         MA90C840
C       * VMULFF - FIRST MATRIX TIMES SECOND MATRIX                      MA90C850
C       * LINV2F - INVERSE OF A MATRIX                                   MA90C860
C                                                                        MA90C870
 17     CONTINUE                                                         MA90C880
        CALL VMULFM(A,W,40,10,40,40,40,C,10,IER1)                        MA90C890
        CALL VMULFF(C,A,10,40,10,10,40,D,10,IER2)                        MA90C900
        CALL VMULFF(C,Z,10,40,1,10,40,E,10,IER3)                         MA90C910
        CALL LINV2F(D,10,10,DINV,3,WKAREA,IER4)                          MA90C920
        CALL VMULFF(DINV,E,10,10,1,10,10,ANS,10,IER5)                    MA90C930
C                                                                        MA90C940
C       PRINT CALCULATED PHASE AND AMPLITUDE WEIGHTS                     MA90C950
C                                                                        MA90C960
```

```
      WRITE(6,999)                                                      MA900970
999   FORMAT(1X,'ANS MATRIX (PHASE AND AMPLITUDE WEIGHTS) FOLLOWS')     MA900980
      WRITE(6,158)(ANS(I8,1),I8=1,10)                                   MA900990
158   FORMAT(F14.7)                                                     MA901000
C                                                                       MA901010
C     APPLYING THESE PHASE AND AMPLITUDE WEIGHTS, THE RESULTING BEAM    MA901020
C     PATTERN IS CALCULATED (REAL AND IMAGINARY PARTS SEPARATE)         MA901030
C                                                                       MA901040
760   CALL VMLLFF(A,ANS, 40,10,1,40,10,Y,40,IER2)                       MA901050
C                                                                       MA901060
C     THE REAL AND IMAGINARY PARTS OF THE RESULTING BEAM PATTERN        MA901070
C     ARE COMBINED TO GIVE ABSOLUTE VALUE.                              MA901080
C                                                                       MA901090
987   I20=I20+1                                                         MA901100
      XX(I20,1)=SQRT(Y(I20,1)*Y(I20,1)+Y(I20+20,1)*Y(I20+20,1))         MA901110
      IF(I20-20)987,654,654                                             MA901120
654   I19=I19+1                                                         MA901130
      X1(I19,1)=200.0+I19*20.0                                          MA901140
      IF(I19-20)654,655,655                                             MA901150
C                                                                       MA901160
C     PRINT SOURCE DEPTH AND ABSOLUTE VALUE OF BEAM PATTERN AT          MA901170
C     THAT DEPTH                                                        MA901180
C                                                                       MA901190
655   WRITE(6,988)(X1(I,1),XX(I,1),I=1,20)                              MA901200
      WRITE(9,988)(X1(I,1),XX(I,1),I=1,20)                              MA901210
988   FORMAT(F6.2,' METER DEPTH HAS STRENGTH OF ',F14.7)                MA901220
172   CONTINUE                                                          MA901230
      STOP                                                              MA901240
      END                                                               MA901250
$ENTRY                                                                  MA901260
                                                                        MA901270
 ANS MATRIX (PHASE AND AMPLITUDE WEIGHTS) FOLLOWS                       MA901280
    -0.0050524                                                          MA901290
     2.2758930                                                          MA901300
     0.5812412                                                          MA901310
    -3.6065380                                                          MA901320
    -4.3302970                                                          MA901330
     0.2429787                                                          MA901340
    -1.3739130                                                          MA901350
    -3.0745440                                                          MA901360
     2.1446630                                                          MA901370
     0.7122661                                                          MA901380
                                                                        MA901390
                                                                        MA901400
                                                                        MA901410
                                                                        MA901420
                                                                        MA901430
                                                                        MA901440
```

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220.00 METER DEPTH HAS STRENGTH OF        0.474306   MA901450
240.00 METER DEPTH HAS STRENGTH OF        0.285057   MA901460
260.00 METER DEPTH HAS STRENGTH OF        0.212856   MA901470
280.00 METER DEPTH HAS STRENGTH OF        0.080418   MA901480
300.00 METER DEPTH HAS STRENGTH OF        0.071669   MA901490
320.00 METER DEPTH HAS STRENGTH OF        0.034932   MA901500
340.00 METER DEPTH HAS STRENGTH OF        0.112468   MA901510
360.00 METER DEPTH HAS STRENGTH OF        0.053037   MA901520
380.00 METER DEPTH HAS STRENGTH OF        0.069397   MA901530
400.00 METER DEPTH HAS STRENGTH OF        0.134308   MA901540
420.00 METER DEPTH HAS STRENGTH OF        0.117223   MA901550
440.00 METER DEPTH HAS STRENGTH OF        0.043795   MA901560
460.00 METER DEPTH HAS STRENGTH OF        0.018534   MA901570
480.00 METER DEPTH HAS STRENGTH OF        0.023968   MA901580
500.00 METER DEPTH HAS STRENGTH OF        0.019799   MA901590
520.00 METER DEPTH HAS STRENGTH OF        0.089187   MA901600
540.00 METER DEPTH HAS STRENGTH OF        0.019037   MA901610
560.00 METER DEPTH HAS STRENGTH OF        0.187521   MA901620
580.00 METER DEPTH HAS STRENGTH OF        0.027198   MA901630
600.00 METER DEPTH HAS STRENGTH OF        0.074082   MA901640
                                                     MA901650
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LIST OF REFERENCES

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3. Melsa, J. L. and Cohn, A. B., Decision and Estimation Theory, McGraw-Hill, 1978.

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Thesis
M272 McVicar
c.1 Amplitude shading and phase weighting of a vertical linear array in the SOFAR channel by the linear minimum variance estimation technique.